FINITE-FREQUENCY IDENTIFICATION: SELFTUNING OF TEST SIGNAL

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Abstract: Linear stable plant with unknown coefficients in the presence of an unknown-but-bounded disturbance is considered. Finite-frequency technique for identification of the plant makes use of a test signal with minimal quantity of harmonics (this value is equal to a plant state-space dimension). It is shown, if frequencies of the test signal are chosen outside a natural frequencies band (where log magnitude of plant has corner frequencies) then identification results may very strongly depend on errors of a frequency characteristics determination. In order to find a estimate of the boundaries of the natural frequencies band a procedure of selftuning of test signal frequencies is given. Test signal amplitudes are selftuned as well. It provides the prescribed boundaries of the input and output plant.

Keywords: Identification, unknown-but-bounded disturbance, frequency response, test signal, self-tuning.

1. INTRODUCTION

In the last decades, various identification methods were developed for control plants subjected to unknown-but-bounded external disturbances and noises. These are the instrumental-variable method (Wong and Polak, 1967;Tsypkin and Poznyak, 1989;Ljung, 1999), finite-frequency identification (Alexandrov, 1994), randomized algorithms (Polyak and Granichin, 2003;Bunich and Bakhtatze,2003). In these methods, a test signal is used which is formed before the identification is initiated, using an *a priori* information about the parameters of the plant, disturbance, and noise. The test signal has to be independent on disturbance and noise and the plant output has to be in the prescribed limits.

Since the form and the parameters of a test signal affect essentially the accuracy and duration of identification, selftuning of this signal in the process of identification is required when *a priori* information is little.

In finite-frequency identification, the test signal is a sum of harmonics with given amplitudes and frequencies; their number is equal to the state-space plant dimension. In order to provide the limits on the input and output plant, selftuning of the amplitudes is required. Selftuning of test frequencies is needed as well, since it is intuitively clear that they have to be chosen from an interval (natural frequency band), where corner frequencies of log magnitude of the plant are located; however this interval is not known. A method of the experimental estimation of the natural frequency band was proposed in (Alexandrov, 2001), which is the basis of a procedure of the frequencies selftuning described here.

The paper consists of two parts. In the first part, it is shown that if the test frequencies are taken outside the natural frequency band, then the results of identification may depend heavily on errors in the determination of the frequency characteristics. In the second part, the procedure of selftuning for test signal is proposed.

The paper is organized as follows. First, the finite-frequency identification technique is described. Next, in Section 3, the problem of test signal selftuning is formulated. Section 4 is devoted to the analysis of sensitivity of the identification results to the choice of test frequencies. Finally, the procedure of test signal selftuning is given in Section 5.

2. PRELIMINARIES

A completely controllable, asymptotically stable plant is described by the following equation:

$$\begin{aligned} d_n y^{(n)} + \cdots + d_1 \dot{y} + y = \\ k_\gamma u^{(\gamma)} + \cdots + k_1 \dot{u} + k_0 u + f, \quad t \geq t_0, \end{aligned} \tag{1}$$

where $y(t)$ is the measured output, $u(t)$ is the input to be controlled, $y^{(i)}$, $u^{(j)}$ $\left(i = \overline{1,n},\ \ j = \overline{1,\gamma}\right)$ are the derivatives of these functions, $f(t)$ is unknown-but-bounded disturbance. The coefficients d_i and k_j $\left(i = \overline{1,n},\ \ j = \overline{0,\gamma}\right)$ are some unknown numbers; n and γ are known, and $\gamma < n$.

The identification problem is to find estimates $\hat{d}_i$ and $\hat{k}_j$ $\left(i = \overline{1,n},\ \ j = \overline{0,\gamma}\right)$ of the plant coefficients such that the identification errors satisfy the following relations:

$$\hat{d}_i \div d_i \leq \varepsilon_i^d, \quad \hat{k}_j \div k_j \leq \varepsilon_j^k \quad i = \overline{1,n} \quad j = \overline{0,\gamma}, \tag{2}$$

where ε_i^d and ε_j^k $\left(i = \overline{1,n},\ \ j = \overline{0,\gamma}\right)$ are given numbers, and the symbol $\div$ means: $a \div b = |a - b|/|b|$ if $b \neq 0$ and $a \div b = |a|$ otherwise.

Let us consider the finite-frequency identification technique, which gives a solution to this problem.

A set of $2n$ numbers

$$\alpha_k = \operatorname{Re} w(j\omega_k), \quad \beta_k = \operatorname{Im} w(j\omega_k), \quad k = \overline{1,n}, \tag{3}$$

where

$$w(s) = \frac{k_\gamma s^\gamma + \cdots + k_0}{d_n s^n + d_{n-1} s^{n-1} + \cdots + 1} \tag{4}$$

is called the *frequency domain parameters* (FDP).

The FDP estimates are determined experimentally as follows: after the plant (1) is excited by the test signal

$$u = \sum_{k=1}^{n} \rho_k \sin \omega_k (t - t_0), \qquad t \geq t_0, \tag{5}$$

where the amplitudes ρ_k $\left(k = \overline{1,n}\right)$ and test frequencies ω_k $\left(k = \overline{1,n}\right)$ are specified positive numbers, its output is fed to the Fourier filters, whose outputs give the following FDP estimates:

$$\begin{aligned} \hat{\alpha}_k &= \alpha_k(\tau) = \frac{2}{\rho_k \tau} \int_{t_F}^{t_F+\tau} y(t) \sin \omega_k (t - t_0)\, dt, \\ & \qquad\qquad\qquad\qquad k = \overline{1,n}, \\ \hat{\beta}_k &= \beta_k(\tau) = \frac{2}{\rho_k \tau} \int_{t_F}^{t_F+\tau} y(t) \cos \omega_k (t - t_0)\, dt, \end{aligned} \tag{6}$$

where τ is a filtering time and $t_F \geq t_0$ is the initial instant for filtering.

In order to formulate conditions on the convergence of the FDP estimates (6) to the true FDP, the following functions are introduced:

$$\begin{aligned} \ell_k^\alpha(\tau) &= \frac{2}{\rho_k \tau} \int_{t_F}^{t_F+\tau} \bar{y}(t) \sin \omega_k (t - t_0)\, dt, \\ & \qquad\qquad\qquad\qquad k = \overline{1,n}, \\ \ell_k^\beta(\tau) &= \frac{2}{\rho_k \tau} \int_{t_F}^{t_F+\tau} \bar{y}(t) \cos \omega_k (t - t_0)\, dt, \end{aligned} \tag{7}$$

where $\bar{y}(t)$ is the "natural" output of the plant when the test signal (5) is absent $(u(t) = 0)$.

Definition 2.1. A disturbance $f(t)$ is called strongly FF-filterability if, for the given numbers δ^α and δ^β, there exists filtering time τ^* such that

$$\begin{aligned} \frac{|\ell_k^\alpha(\tau^*)|}{|\alpha_k(\tau^*)|} \leq \delta^\alpha, \quad \frac{\left|\ell_k^\beta(\tau^*)\right|}{|\beta_k(\tau^*)|} \leq \delta^\beta, \\ k = \overline{1,n}, \quad \tau \geq \tau^*, \end{aligned} \tag{8}$$

Conditions (8) can be examined by experiment.

If the disturbance $f(t)$ is strongly FF-filterability, then the filtering errors $\Delta\alpha_k(\tau) = \alpha_k - \alpha_k(\tau)$, $\Delta\beta_k(\tau) = \beta_k - \beta_k(\tau)$ $\left(k = \overline{1,n}\right)$ have the following properties: $\lim_{\tau\to\infty} \Delta\alpha_k(\tau) = \lim_{\tau\to\infty} \Delta\beta_k(\tau) = 0$ $\left(k = \overline{1,n}\right)$.

The estimates of the plant coefficients are found on the basis of the FDP estimates. In fact, the identity $w(s) = \dfrac{k(s)}{d(s)}$ and expressions (3) give the following system of the linear algebraic equations:

$$\hat{k}(s_k) - (\alpha_k + j\beta_k)\hat{\tilde{d}}(s_k) = \alpha_k + j\beta_k \quad k = \overline{1,n}, \tag{9}$$

where $\hat{\tilde{d}}(s) = \hat{d}(s) - 1 = \hat{d}_n s^n + \cdots + \hat{d}_1 s$, $\hat{k}(s) = \hat{k}_\gamma s^\gamma + \cdots + \hat{k}_1 s + \hat{k}_0$, $s_k = j\omega_k$ $\left(k = \overline{1,n}\right)$.

Assertion 2.1. If the plant (1) is completely controllable, then system (9) has a unique solution d_i, k_j $\left(i = \overline{1,n},\ \ j = \overline{0,\gamma}\right)$ which does not depend on the choice of the frequencies ω_i $(\omega_i \neq \omega_j$ $(i \neq j)$, $\omega_i \neq 0$ $\left(i = \overline{1,n}\right))$.

Substituting the FDP by their estimates, the following *frequency equations* of identification

$$\hat{k}(s_k) - (\hat{\alpha}_k + j\hat{\beta}_k)\hat{\hat{d}}(s_k) = \hat{\alpha}_k + j\hat{\beta}_k \quad k = \overline{1,n}, \qquad (10)$$

are obtained.

In order to examine requirements (2), the frequency techniques of model validation (Alexandrov, 1999) may be used.

3. PROBLEM STATEMENT

Above it has been assumed that the amplitudes and frequencies of test signal (5) are given. In order to specify of them a priori, it need a large volume of information about plant (1). In fact, first,the plant output and input are bounded by given numbers y^* and u^*:

$$|y(t)| \leq y^*, \quad |u(t)| \leq u^*, \quad t \geq t_0, \qquad (11)$$

where y^* such that

$$|\bar{y}(t)| < y^*, \quad t \geq t_0. \qquad (12)$$

The conditions (11) are provided by a choice of the amplitudes ρ_k $\left(k = \overline{1,n}\right)$ signal (5).

Second, it is intuitively clearly that test frequencies have to be chosen from a frequencies interval, where corner frequencies of log magnitude of the plant (a natural frequencies band), are placed. At first glance, it contradicts the assertion 2.1, however, the assertion describes properties of the equations (9) but for identification the frequency equations (10), where the FDP's are substituted by their estimations, are used. In order to introduce a notion of a natural frequencies band, transfer function of plant (1) is represented as

$$w(s) = k\frac{\prod_{i=1}^{p_1}(s+\omega_{1,i})\prod_{i=1}^{p_2}\left(s^2+2\xi_{i,2}\omega_{i,2}+\omega_{i,2}^2\right)}{\prod_{i=1}^{p_3}(s+\omega_{3,i})\prod_{i=1}^{p_4}\left(s^2+2\xi_{i,4}\omega_{i,4}+\omega_{i,4}^2\right)}. \qquad (13)$$

A set $L = \{\, |\omega_{1,1}|, |\omega_{1,2}|, \ldots, |\omega_{1,p_1}|; |\omega_{2,1}|, |\omega_{2,2}|, \ldots, |\omega_{2,p_2}|; \omega_{3,1}, \omega_{3,2}, \ldots, \omega_{3,p_3}; \omega_{4,1}, \ldots, \omega_{4,p_4}\}$ is called a *natural frequencies* of the plant (1).

The *lower* (ω_l) and *upper*(ω_u) boundaries of the natural frequencies are marked as

$$\omega_l = \min L \qquad \text{and} \qquad \omega_u = \max L.$$

Denote $\Omega_l = \{\omega : \omega \in (0, \omega_l)\}$,
$\Omega = \{\omega : \omega \in [\omega_l, \omega_u]\}$, $\Omega_u = \{\omega : \omega \in (\omega_u, \infty)\}$.

In the next section, it is shown that, if the test frequencies are taken from low-frequencies band $\left(\omega_k \in \Omega_l,\, k = \overline{1,n}\right)$ or upper-frequencies band $\left(\omega_k \in \Omega_u,\, k = \overline{1,n}\right)$, then small errors of the filtration may give large errors of identification and therefore a part of the test frequencies have to lie into the natural frequency band $\left(\omega_k \in \Omega,\, k \in \overline{1,n}\right)$.

Problem 3.1. Find a way of the amplitudes and frequencies self-tuning of the test signal (5) such that the plant output and input satisfy the requirements (11) and a part of the test frequencies were into the natural frequencies band $\left(\omega_k \in \Omega,\, k \in \overline{1,n}\right)$.

A solution of the problem is based on the following assertion (Alexandrov,2001).

Assertion 3.1. Let plant (1) be exited by the test signal $u(t) = \rho_1 \sin\omega_1(t - t_0)$ and its output is fed to the Fourier's filter (6) $(n = 1)$. There exist a sufficiently large filtering time $\tau = \tau^*$ and a sufficiently small test frequency $\omega_1 \in \Omega_l$ such that a number

$$\bar{\omega}_l(\tau^*) = \left|\frac{\omega_1\alpha_1(\tau^*)}{\beta_1(\tau^*)}\right| \qquad (14)$$

is nearly to the lower (ω_l) boundary of natural frequencies (the nearness depends on ω_1, τ^* and a difference of ω_l and a natural frequency that is nearest to ω_l).

A analogous assertion is proved for the estimate of the upper boundary $(\bar{\omega}_u)$ of the natural frequencies band.

4. SENSITIVITY ANALYSIS OF IDENTIFICATION ERRORS

Denote the maximal relative errors of filtration and identification as

$$\eta_{\alpha,\beta} = \max_{1\leq k\leq n}\left\{\hat{\alpha}_k \div \alpha_k\,,\, \hat{\beta}_k \div \beta_k\right\} \qquad (15)$$

$$\eta_{d,k} = \max_{1\leq k\leq n}\left\{\hat{d}_k \div d_k\,,\, \hat{k}_k \div k_k\right\} \qquad (16)$$

respectively.

Definition 4.1. Number $C = \dfrac{\eta_{dk}}{\eta_{\alpha\beta}}$ is called a sensitivity coefficient of identification errors with respect to filtration errors.

Assertion 4.1. There exists a set of the test frequencies $\omega_k \in \Omega_l$ $\left(k = \overline{1,n}\right)$ and the strongly FF-filterability disturbance $f(t)$ such that the sensitivity coefficient C is greater than any given positive number C^* $(C > C^*)$.

A proof of this assertion is based on two assertions (properties) and a lemma that are formulated below.

Using transfer function (4) the following expression for the FDP is obtained

$$\alpha_k = \frac{\sum_{q=0}^{[\frac{n+\gamma}{2}]} l_{2q}\omega_k^{2q}}{\sum_{q=0}^{n} m_q\omega_k^{2q}}, \quad \beta_k = \frac{\sum_{q=0}^{\{\frac{n+\gamma}{2}\}-1} l_{2q+1}\omega_k^{2q+1}}{\sum_{q=0}^{n} m_q\omega_k^{2q}}, \tag{17}$$

$\left(k=\overline{1,n}\right)$, where $\ell_0 = k_0$, $\ell_1 = k_1 - k_0 d_1$, ..., $[\cdot]$ and $\{\cdot\}$ are integer numbers nearest to $\cdot$ such that $[\cdot] \le \cdot \le \{\cdot\}$.

The FDP (17) can be approximated by

$$\alpha_k^l = l_0, \quad \beta_k^l = \sum_{q=0}^{\{\frac{n}{2}\}-1} l_{2q+1}\omega_k^{2q+1}, \quad k=\overline{1,n}. \tag{18}$$

The following assertion is almost obviously.

Property 4.1. For any small number $\delta_l > 0$ there exists a set of the test frequencies $\omega_k \in \Omega_\ell$ $\left(k=\overline{1,n}\right)$ such that

$$\alpha_k^l \div \alpha_k < \delta_l, \quad \beta_k^l \div \beta_k < \delta_l \quad k=\overline{1,n}. \tag{19}$$

■

The FDP estimates may be represented as

$$\begin{aligned} \hat{\alpha}_k &= \alpha_k^l + \varepsilon_\alpha^l(\omega_k) + \Delta\alpha_k(\tau), \\ \hat{\beta}_k &= \beta_k^l + \varepsilon_\beta^l(\omega_k) + \Delta\beta_k(\tau), \end{aligned} \tag{20}$$

where $\varepsilon_\alpha^l(\omega_k) = \alpha_k - \alpha_k^l$ and $\varepsilon_\beta^l(\omega_k) = \beta_k - \beta_k^l$ $\left(k=\overline{1,n}\right)$.

Lemma 4.1. For the specified test frequencies ω_k $\left(k=\overline{1,n}\right)$ there exists a strongly FF-filterability disturbance $f(t)$ and filtration time τ^* such that the following equalities

$$\begin{aligned} \varepsilon_\alpha^l(\omega_k) &= -\Delta\alpha_k(\tau^*) \\ \varepsilon_\beta^l(\omega_k) &= -\Delta\beta_k(\tau^*) \end{aligned} \quad k=\overline{1,n} \tag{21}$$

hold. ■

The lemma proof is given in Appendix.

The equalities

$$\hat{\alpha}_k = \alpha_k^l, \quad \hat{\beta}_k = \beta_k^l, \quad k=\overline{1,n}, \tag{22}$$

follow (20), where $\tau = \tau^*$, and conditions (21).

Property 4.2. If the FDP estimates have the view (22) then the solution of frequency equations (10) is unique and it has the following view

$$\hat{d}_i = 0, \quad i=\overline{1,n}. \tag{23}$$

Proof of this property is bulky (for shortness it is omitted), but its idea may be explained by an example when $n=2$. In this case the frequency equations (10) are rewritten as

$$\begin{aligned} \hat{k}_0 + \jmath\omega_k\hat{k}_1 - (\hat{\alpha}_k + \jmath\hat{\beta}_k)(\jmath\omega_k\hat{d}_1 - \omega_k^2\hat{d}_2) &= \\ = \hat{\alpha}_k + \jmath\hat{\beta}_k, \quad k=\overline{1,2}. \end{aligned} \tag{24}$$

In accordance with expression (18) the FDP estimates $\hat{\alpha}_k = l_0$ and $\hat{\beta}_k = l_1\omega_k$ $\left(k=\overline{1,2}\right)$ and then the system has the obvious solution $\hat{d}_1 = \hat{d}_2 = 0$, $\hat{k}_0 = l_0$, $\hat{k}_1 = l_1$. This solution is unique since the determinant of the system is equal to $[l_1(\omega_1 - \omega_2)(\omega_1+\omega_2)]^2$ and it is not zero.

Now using properties 4.1 and 4.2 and lemma 4.1 the assertion 4.1 can be easily proved. In fact, let any large number C^* be specified. Take $\delta_l = 1/C^*$ and find the frequencies $\omega_k \in \Omega_l$ $\left(k=\overline{1,n}\right)$ for which the inequalities (19) are fulfilled.

The equalities (23) give $\eta_{dk} \ge 1$. On the other hand, the expressions(19) and (22) give $\eta_{\alpha\beta} < \delta_l$ and therefore the assertion 4.1 is proved.

5. PROCEDURE OF SELFTUNING

During process of selftuning of the test signal, the plant (1) is exited by the following test signal

$$\begin{gathered} u(t) = \rho_{[i]}^{[j]} \sin\omega_{[i]}(t-t_0), \\ t_{[i]}^{[j-1]} \le t < t_{[i]}^{[j]}, \quad t_{[i+1]}^{[0]} = t_{[i]}^{[n_{[i-1]}]}, \\ i=\overline{1,n_\omega} \quad j=\overline{1,n_{[i]}}, \end{gathered} \tag{25}$$

where i $\left(i=\overline{1,n_\omega}\right)$ is a number of a tuning frequency interval, j $\left(j=\overline{1,n_{[i]}}\right)$ is a number of a tuning amplitude subinterval.

Durations of all subintervals are equal

$$T_{[i]} = t_{[i]}^{[j]} - t_{[i]}^{[j-1]} = \frac{2\pi}{\omega_{[i]}} p_{[i]}, \quad i=\overline{1,n_\omega},$$

where $p_{[i]}$ $\left(i=\overline{1,n_\omega}\right)$ are given numbers.

Procedure 5.1

(1) Feed to the plant (1) signal (25) with a given sufficiently small frequency $\omega_{[1]} = \omega^*$ and an amplitude $\rho_{[1]}^{[0]} = u^*$; examine the first condition (11). If it is satisfied,the searched amplitude is found. Otherwise, put $\rho_{[1]}^{[1]} = u^*/\delta$, where $\delta > 1$ is a given number, and

so on until the condition (11) is satisfied for $\rho_{[1]}^{[n_{[1]}]} = \rho^*$.

Measure the outputs $\alpha_1(\tau^*)$ and $\beta_1(\tau^*)$ of the Fourier's filter (6), where $n = 1$, $\rho_1 = \rho^*$, $\omega_1 = \omega_{[1]}$ (a way of determination of filtering time (τ^*) from the conditions (8) of the strongly FF-filterability is given after the procedure).

(2) Calculate the lower boundary estimate $\bar{\omega}_l(\tau^*)$ by formulae (14).

(3) Repeat the operations 1-2, putting in the signal (25): $\omega_{[2]} = \omega_{[1]}/\delta_\omega$, where $\delta\omega > 1$ is a given number and find new lower boundary estimate $- \omega_l(\tau^{**})$; examine the condition

$$\hat{\bar{\omega}}_l(\tau^*) \div \hat{\bar{\omega}}_l(\tau^{**}) \leq \varepsilon_\omega, \tag{26}$$

where ε_ω is a given sufficiently small number.

If inequality (26) is satisfied then the searched $\hat{\bar{\omega}}_l = \bar{\omega}_l(\tau^{**})$. Otherwise, put $\omega_{[3]} = \omega_{[2]}/\delta_\omega$ and so on until the inequality is satisfied.

(4) Repeat the operations 1-3 for a sufficiently large frequency $\omega_{[1]}$ and find the upper boundary estimate $\hat{\bar{\omega}}_u$ of the natural frequencies of the plant.

(5) Choice the n frequencies of a set $\hat{\bar{\Omega}} = \{\omega : \omega \in [\hat{\bar{\omega}}_l, \hat{\bar{\omega}}_u]\}$ (for example, calculate of them as $\omega_1 = \hat{\bar{\omega}}_l$, $\omega_k = \hat{\bar{\omega}}_l + \frac{\hat{\bar{\omega}}_u - \hat{\bar{\omega}}_l}{n-1}(k-1)$ $(k = \overline{2,n})$); repeat operation 1 for each frequency ω_k $(k = \overline{1,n})$; find the FDP estimates $\hat{\alpha}_k$, $\hat{\beta}_k$ $(k = \overline{1,n})$; solve the frequency equations (10), that gives the plant coefficients estimations.

Form the vector L of the natural frequencies of the identified plant, find a set of these frequencies Ω^{id}. If $\omega_k \in \Omega^{id}$ $(k = \overline{1,n})$ than the selftuning is ended. Otherwise, repeat the procedure 5.1, decreasing the numbers δ^α, δ^β and ε_ω, until the requirement $\omega_k \in \Omega^{id}$ $(k = \overline{1,n})$ is satisfied.

In order to find the filtering time τ^* for operation 1, this operation is formed from the pause-intervals, where $\rho_{[i]}^{[j]} = 0$, and the test-intervals, where $\rho_{[i]}^{[j+1]} = \rho^*$.

Let $\tau^* = T_{[1]}$. Using the outputs of filters (6) and (7), the inequalities (8), in which δ^α and δ^β are given numbers, are examined. If they are satisfied than $\tau^* = T_{[1]}$. Otherwise, the operation 1 is repeated for $\tau^* = T_{[1]}\delta_T$, where $\delta_T > 1$ is a given number, and so on until the conditions (8) are satisfied.

Remark 5.1 The described way of finding of the filtration time τ^* serves for an experimental test of the a priori assumption about the strongly FF-filtrability of the disturbance $f(t)$. If the filtering time τ^*, for which the conditions (8) satisfy, does not exist then the frequency $\omega_{[1]}$ is changed until the conditions (8) are satisfied.

Let us introduce a class of disturbances $f(t)$ for that convergence of procedure 5.1 is readily proved.

Definition 5.1. The disturbance $f(t)$ is contiguously stationary if

$$\max_{t_{[i]}^{[j-1]}+t_F \leq t \leq t_{[i]}^{[j]}} |\bar{y}(t)| \div \max_{t_{[i]}^{[j]}+t_F \leq t \leq t_{[i]}^{[j+1]}} |\bar{y}(t)| \leq \varepsilon_y, \tag{27}$$

where ε_y is a given sufficiently small number.

The following assertion is almost obviously.

Assertion 5.1. If the disturbance $f(t)$ is contiguously stationary and the strongly FF-filterability then procedure 5.1 converges to the frequencies $\omega_k \in \Omega^{id}$ $(k = \overline{1,n})$ and the requirements (11) to the input and output plant are satisfied.

Remark 5.2. The number ε_y may be essentially enhanced, if the amplitude of signal (25) is tuned, when the filtration time τ^* is searched.

MATLAB-function "Finite-frequency identification" was created (Alexandrov and Orlov, 2005) on the base of the procedure 5.1. Applications of this function show its effectiveness.

6. CONCLUSION

In this paper is shown that, if the frequencies of the test signal (5) are chosen outside of the natural frequencies band of plant (1), then the sensitivity of identification results to filtration errors may be very high (assertion 4.1).

In connection with it, the finite-frequency method is added by the procedure 5.1. This procedure (by selftuning of the test frequencies) gives the part of the test frequencies into the natural frequencies band. In addition, in order to carry out the requirements (11) to boundaries of the input and output plant, the selftuning of the test signal amplitudes is proposed.

This development of the finite-frequency method gives new possibilities for identification of the real plants.

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APPENDIX

A.1 PROOF OF LEMMA 4.1

Let disturbance $f(t)$ have the form

$$f(t) = \sum_{k=1}^{2n} \rho_k^f \sin \omega_k^f (t - t_0), \quad t \geq t_0, \tag{A.1}$$

where ρ_k^f and ω_k^f $\left(k = \overline{1, 2n}\right)$ are unknown numbers that should be determined from conditions (21) and inequalities

$$\omega_i^f \neq \omega_j \quad i = \overline{1, 2n} \quad j = \overline{1, n}, \tag{A.2}$$

which signify that disturbance (A.1) is strong FF-filterable, amplitudes ρ_k^f $\left(k = \overline{1, 2n}\right)$ satisfy the following inequalities

$$\sum_{k=1}^{2n} |\rho_k^f| \leq f^* \tag{A.3}$$

where f^* is given number Filtration errors have the following structure

$$\begin{aligned} \Delta\alpha_k(\tau) &= e_k^\alpha(\tau) + \ell_k^\alpha(\tau), \\ \Delta\beta_k(\tau) &= e_k^\beta(\tau) + \ell_k^\beta(\tau), \end{aligned} \quad k = \overline{1, n}, \tag{A.4}$$

where $e_k^\alpha(\tau)$ and $e_k^\beta(\tau)$ are vanished functions.

In order to find functions $\ell_k^\alpha(\tau)$ and $\ell_k^\beta(\tau)$ $\left(k = \overline{1, n}\right)$ function $\bar{y}(t)$ is necessary. To this effect the equation (1) rewritten in state space as

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + \boldsymbol{\psi} f, \quad \bar{y} = \boldsymbol{c}^T \boldsymbol{x}, \tag{A.5}$$

where A is a matrix, $\boldsymbol{\psi}$, $\boldsymbol{c}$ are vectors.

The solution of these equations under condition (A.1) is

$$\begin{aligned} \bar{y}(t) = \sum_{k=1}^{2n} \rho_k^f \Big[\alpha_k^f \sin \omega_k^f (t - t_0) + \\ + \beta_k^f \cos \omega_k^f (t - t_0) + æ\left(t, \omega_k^f\right)\Big] + æ^0(t), \end{aligned} \tag{A.6}$$

where $\alpha_k^f = \operatorname{Re} w^f(j\omega_k^f)$, $\beta_k^f = \operatorname{Im} w^f(j\omega_k^f)$ $\left(k = \overline{1, 2n}\right)$, $w^f(s) = \boldsymbol{c}^T(Es - A)^{-1}\boldsymbol{\psi}$,

$$\begin{aligned} æ\left(t, \omega_k^f\right) &= \boldsymbol{c}^T e^{A(t-t_0)} \operatorname{Im}(j\omega_k^f - A)^{-1}\boldsymbol{\psi}, \\ æ^0(t) &= \boldsymbol{c}^T e^{A(t-t_0)} \boldsymbol{x}(t_0). \end{aligned} \tag{A.7}$$

Substituting function (A.6) into the expression (7) and taking account (A.4) and equalities (21) the following system of the linear algebraic equations for determination of the amplitudes ρ_k^f $\left(k = \overline{1, 2n}\right)$ is derived

$$\begin{aligned} \sum_{i=1}^{2n} q_{ki}^\alpha(\tau^*)\rho_i^f &= -\varepsilon_\alpha^l(\omega_k) - e_k^\alpha(\tau^*) - q_{k0}^\alpha(\tau^*) \\ & \qquad k = \overline{1, n}, \\ \sum_{i=1}^{2n} q_{ki}^\beta(\tau^*)\rho_i^f &= -\varepsilon_\beta^l(\omega_k) - e_k^\beta(\tau^*) - q_{k0}^\beta(\tau^*) \end{aligned} \tag{A.8}$$

where $q_{k0}^\alpha(\tau^*)$ and $q_{k0}^\beta(\tau^*)$ are vanished functions.

Choosing a filtration time τ^* and the disturbance frequencies ω_i^f $\left(i = \overline{1, 2n}\right)$ from interval $(0, \infty)$, when the test frequencies ω_k and amplitudes ρ_k $\left(k = \overline{1, n}\right)$ are specified, a unique solution of system (A.8) may be always obtained. For the condition (A.3) to fulfill the numbers $\varepsilon_\alpha^l(\omega_k)$ and $\varepsilon_\beta^l(\omega_k)$ in the right parts of equations (A.8) must be decreased by a decrease of the frequencies ω_k $\left(i = \overline{1, n}\right)$. It is easily shown that the coefficients of the left parts of this equations are almost independent on the frequencies ω_k $\left(i = \overline{1, n}\right)$ if frequencies ω_i^f $\left(i = \overline{1, 2n}\right)$ puts sufficiently large.