Finite-frequency identification of plant with time delay

Alexandrov Albert∗ Yuri Orlov∗∗ Palenov Maxim∗∗∗

∗ Institute of Control Sciences RAS, Profsoyuznaya 65, Moscow, Russia (e-mail: alex7@ipu.rssi.ru)
** Electrostal Polytechnic Institute of National Technological University, Pervomayskaya, 7, Electrostal, Russia, 144000, (e-mail: yu_orlov@mail.ru)
*** Institute of Control Sciences RAS, Profsoyuznaya 65, Moscow, Russia (e-mail: max_elek@mail.ru)

Abstract: A method of finite-frequency identification for stable plants with time-delay in the presence of unknown-but-bounded disturbance and measurement noise is proposed. This method uses test signal which is a sum of harmonics. Quantity of harmonics does not exceed count of plant coefficients. Conditions of convergence of identification process are also given.

1. INTRODUCTION

At present, the control theory has at its disposal a number of identification methods for plants specified by linear differential equations with time delay. In some sense these methods fall into two categories depending on the assumptions on the measurement errors and exogenous disturbances affecting the plant.

The methods of the first class deal with the plants subjected to disturbances of the stochastic nature; i.e., random processes having known statistical characteristics. These are various versions of the method of least squares and the stochastic approximation method; e.g., see well-known monographs as L. [1999].

The second class comprises the identification methods under unknown-but-bounded disturbances (whose statistical properties are not known) such as randomized algorithms Granichin O.N. and Polyak B.T. [2003] and Finite-frequency identification, see A.G. [1994] and A.G. [1999].

A somewhat specific position is occupied by the method of instrumental variables Wong K.Y. and Polak E. [1967]. It is developed in the framework of the first class; however, in contrast to the other methods in the class, it is applicable to the problems of the second class so that it is reasonable to consider it as a method of the second category.

The identification process can be passive or active forms. In the passive identification, the measured input of the plant has the meaning of a control action which depends on the control objectives and is not related to identification of the plant. With such an input, identification might not be possible; hence, active identification is often practiced where, in addition to control, the measured input contains an extra component, a so-called test signal aimed at identifying the plant.

The finite-frequency identification method was designed for the needs of active identification. The test signal is represented by the sum of harmonics with automatically tuned (self-tuned) amplitudes and frequencies where the number of harmonics does not exceed the state space dimension of the plant. The advantage of the method of finite-frequency identification, compared to other existing methods, is that the method converges regardless of the type and intensity of the external disturbance. There is the approach (See G. [1991]), in which the method can be used when the external disturbance contains signals with frequencies of the test signal. Amplitudes and frequencies of test signal are affected to the speed of convergence of identification. The choice of frequency and amplitude of the test signal is the main issue in this area and there are many articles that solve it (See, example A.G. [1999] and A.G. [2005]).

Along with this there are many papers where methods of identification of plant with time-delay are proposed. These methods works in the assumption that we know the static characteristics of the external disturbance. A nice review of the time-delay estimation methods is given in Bjorklund S. and Ljung L. [2003]. Also interesting recommendations are given on how to properly choose the estimation method and the identification input sequence from a simulation oriented evaluation. The adaptive observer, that can estimate the coefficients of the plant and the time-delay, proposed in M’Saad M. and Farza M. [2009], Yamaguchi H. et al. [2011]. The process identification from sinusoidal test data by estimating step response described in Ahmed S. et al. [2009]. The adaptive control scheme for uncertain time-delay systems with unknown polyharmonic external disturbance is proposed in Pyrkin A. et al. [2010a,b], Bobtsov A. et al. [2010]. For case with unknown-but-bounded disturbances the modified method of instrumental variables, that can estimate the coefficients of the plant and the time-delay, proposed in Baysse A. et al. [2011]. However, this method works with sampled data that stored during the experiment.

In this paper, we propose a method of finite-frequency identification of plants with time-delay. The problems of self-tuning of test signal are solved for plants without time-delay in A.G. [2005] and may be used in this case.
The paper is organized as follows. In the second section the problem statement is given. Then, frequency equations of identification which connect some points of Nyquist plot of plant with plant coefficients are introduced in third section. A formulae for time-delay is given in fourth section. The section 5 describes conditions of convergence of identification. The identification algorithm is given in section 6. And finally an example that demonstrates the proposed identification method is described in section 7.

2. PROBLEM STATEMENT

A completely controllable, asymptotically stable plant with time-delay is described by the following equation:

\[ d_n y^{(n)}(t) + \cdots + d_1 y(t) + y(t) = k_m u^{(m)}(t - \tau) + \cdots + k_0 u(t - \tau) + f(t), \]

where \( y(t) \) is the output plant, \( u(t) \) is the input to be controlled, \( \tau \) is the time-delay \( (\tau > 0) \); \( f(t) \) is unknown but-bounded disturbance:

\[ |f(t)| \leq f^*, \]

where \( f^* \) is some positive number.

Coefficients \( d_n, k_\mu (\nu = \overline{1,n}, \mu = \overline{0,m}) \) and the time-delay \( \tau \) aren’t known. Roots of polynomial \( k(s) \) lie into left half-plane.

The measured output of plant has the following form:

\[ \hat{y}(t) = y(t) + \eta(t), \]

where \( \eta(t) \) is the measurement noise.

The test signal is the sum of harmonics

\[ u(t) = \sum_{i=1}^{l} \rho_i \sin \omega_i(t) \]

where count of harmonics is \( l = n + m + 1 \); the amplitudes \( \rho_i > 0 \) and frequencies \( \omega_i (i = \overline{1,l}) \) are given positive numbers, which may be self-tuned by using proposed in A.G. [2005] algorithm.

The frequencies \( \omega_i (i = \overline{1,l}) \) should satisfy following condition

\[ \omega_i \neq \omega_j \ (i,j = \overline{1,l}, i \neq j). \]

The identification problem is to find estimate \( \hat{\tau} \) of time-delay and estimates \( \hat{d}_\nu \) and \( \hat{k}_\mu (\nu = \overline{1,n}, \mu = \overline{0,m}) \) of the plant coefficients.

Remark 1. Some technical devices are not able to realize the test signal as a sine or a sum of sines, in this case is possible to use the sum of meanders as a test signal. However, this approach will increase the time of identification.

3. PLANT COEFFICIENTS IDENTIFICATION

In this section frequency equations of identification which connect points of Nyquist plot of plant at frequencies \( \omega_i (i = \overline{1,l}) \) with plant coefficients \( d_\nu, k_\mu (\nu = \overline{1,n}, \mu = \overline{0,m}) \) of the plant (1) are introduced.

After Laplace transform the equation (1) has a following form

\[ d(s)Y(s) = k(s)e^{-\gamma t}U(s) + F(s), \]

with

\[ d(s) = d_n s^n + d_{n-1}s^{n-1} + \cdots + d_1 s + 1, \]

\[ k(s) = k_m s^m + k_{m-1}s^{m-1} + \cdots + k_1 s + k_0, \]

where \( U(s) \) and \( Y(s) \) are the input and the output of the plant, \( F(s) \) is the external disturbance.

Let us introduce numbers

\[ \alpha_i = \Re w_r(j\omega_i), \ \beta_i = \Im w_r(j\omega_i), \ i = \overline{1,l}, \]  

where

\[ w_r(j\omega_i) = w(j\omega_i)e^{-j\omega_i \tau} = \frac{k(j\omega_i)}{d(j\omega_i)}e^{-j\omega_i \tau}, \ i = \overline{1,l}. \]

that are called frequency domain parameters (FDP) A.G. [1994] of plant (1).

The FDP estimates are determined experimentally by Fourier filters

\[ \hat{\alpha}_i = \alpha_i(T) = \frac{2}{\rho_1} \int_{T_p}^{T_p + \pi} \tilde{y}(t) \sin \omega_i t dt, \ i = \overline{1,l}, \]

\[ \hat{\beta}_i = \beta_i(T) = \frac{2}{\rho_1} \int_{T_p}^{T_p + \pi} \tilde{y}(t) \cos \omega_i t dt, \]

where \( T_p \) is the initial time moment for filtering, \( T \) - is a filtering time.

Conditions for convergence of the filters (7) to the FDP (5) are given in Section 5.

Equations relating the FDP \( \alpha_i, \beta_i \ (i = \overline{1,l}) \) with the coefficients of the plant \( k_\mu (\mu = \overline{1,m}) \ d_\nu (\nu = \overline{1,n}) \), have the form

\[ k(j\omega_i)k(-j\omega_i) - \gamma_i [d(j\omega_i)d(-j\omega_i) - 1] = \gamma_i, \]

\[ \gamma_i = \alpha_i^2 + \beta_i^2 \]

where

\[ k(j\omega_i)k(-j\omega_i) = \sum_{\mu=0}^{m} \bar{k}_\mu(-1)^\mu \omega^{2\mu}, \]

\[ d(j\omega_i)d(-j\omega_i) - 1 = \sum_{\nu=1}^{n} \bar{d}_\nu(-1)^\nu \omega^{2\nu} \]

are polynomials of even degrees.

Equations (8) is easily obtained. Let’s write (6) as

\[ k(j\omega_i) = w_r(j\omega_i)e^{-j\omega_i \tau}d(j\omega_i), \ i = \overline{1,l}, \]

then

\[ k(j\omega_i)k(-j\omega_i) = w_r(j\omega_i)w_r(-j\omega_i)d(j\omega_i)d(-j\omega_i), \ i = \overline{1,l}. \]

Using the notation (5) and (9) we obtain (8).

Substituting (10) into (8) we obtain frequency equations of identification

\[ \sum_{\mu=0}^{m} (-1)^\mu \omega_i^{2\mu} \bar{k}_\mu = \gamma_i \sum_{\nu=1}^{n} (-1)^\nu \omega_i^{2\nu} \bar{d}_\nu = \gamma_i, \ i = \overline{1,l}. \]

By using A.G. [1994], we conclude that the solution of (11) exists and is unique if the polynomials \( d(s) \) and \( k(s) \) do not contain common roots.

The solution of equations (11) are coefficients \( \bar{d}_\nu \) and \( \bar{k}_\mu \). Let’s introduce follow polynomials
\[ \tilde{d}(s) = \sum_{\nu=1}^{n} \tilde{d}_{\nu} s^{2\nu} + 1 = d(s)d(-s) = \]
\[ = d_n^2 \prod_{\nu=1}^{n} (s - s[d]_{\nu}) (s + s[d]_{\nu}), \]
\[ \tilde{k}(s) = \sum_{\mu=0}^{m} \tilde{k}_{\mu} s^{2\mu} = \ldots \]

The functions \( f(t) \) and \( \eta(t) \) are expanded into a Fourier series, this functions can be written as

\[ f(t) = \sum_{r=1}^{n} a_r \sin \omega_f^r t, \]
\[ \eta(t) = \sum_{p=1}^{n} b_p \sin \omega_p^r t. \]
where \( n_1, a_r, \omega'_f (r = 1, n_2) \) and \( n_2, b_p, \omega'_p (p = 1, n_1) \) are unknown numbers.

If the functions \( f(t) \) and \( \eta(t) \) does not contain frequencies that are coincide with frequencies of the test signal \( \omega'_f \neq \omega_i, \omega'_p \neq \omega_i, r = 1 \rightarrow n_2, i = 1 \rightarrow l, \) then this functions are strongly FF-filterable it is easy to show that

\[
\begin{align*}
\lim_{\mathcal{T} \to \infty} \sigma^2_1(\mathcal{T}) &= 0, \quad \lim_{\mathcal{T} \to \infty} \sigma^2_1(\mathcal{T}) = 0, \\
\lim_{\mathcal{T} \to \infty} \alpha_i(\mathcal{T}) &= \alpha_i, \quad \lim_{\mathcal{T} \to \infty} \beta_i(\mathcal{T}) = \beta_i, \quad i = 1 \rightarrow l,
\end{align*}
\]

When the frequencies of the test signal is coincided with frequencies of functions \( f(t) \) and \( \eta(t) \) then

\[
\begin{align*}
\lim_{\mathcal{T} \to \infty} \sigma^2_1(\mathcal{T}) &= \epsilon^2_i, \quad \lim_{\mathcal{T} \to \infty} \sigma^2_1(\mathcal{T}) = \epsilon^2_i, \\
\lim_{\mathcal{T} \to \infty} \alpha_i(\mathcal{T}) &= \alpha_i + \xi^a_i, \quad \lim_{\mathcal{T} \to \infty} \beta_i(\mathcal{T}) = \beta_i + \xi^b_i, \quad i = 1 \rightarrow l,
\end{align*}
\]

where \( \epsilon^2_i, \xi^a_i, \xi^b_i, \) \( i = 1 \rightarrow l, \) are some positive numbers.

If the numbers \( \epsilon_i, \epsilon^a_i, \) and \( \epsilon^b_i \) \( i = 1 \rightarrow l, \) are sufficiently small then the functions \( f(t) \) and \( \eta(t) \) are called FF-filterable.

6. IDENTIFICATION ALGORITHM

An identification algorithm consist of the following steps.

1. Choice of frequencies \( \omega_i \) and amplitudes \( \rho_i (i = 1, l) \) of the test signal (3), or use algorithm, proposed in A.G. [2005], for self-tuning frequencies and amplitudes.

2. Feed to the plant (1) the test signal (3).

3. The Fourier filters (7) is fed to the measured output of plant \( \hat{y}(t) \). Outputs of the Fourier filters, for given filtering time \( \mathcal{T} \), give the FDP estimates \( \hat{\alpha}_i \) and \( \hat{\beta}_i \) \( i = 1 \rightarrow l, \).

4. Compute the estimates of plant coefficients \( \hat{d}_\nu \) and \( \hat{k}_\mu \) \( (\nu = 1 \rightarrow n, \mu = 1 \rightarrow m) \), by substituting the estimates of the FDP into the formulas (9) and (11).

5. Form the polynomials \( \hat{d}(s) = \sum_{\nu = 1}^{n} \hat{d}_\nu s^{2\nu} + 1 \) and \( \hat{k}(s) = \sum_{\mu = 0}^{m} \hat{k}_\mu s^{2\mu} \) by using estimates, compute its roots \( \pm s^{[d]}_\nu \) \( \nu = 1 \rightarrow n, \mu = 1 \rightarrow m \) and form the polynomials \( \hat{d}(s) = \sum_{\nu = 1}^{n} \hat{d}_\nu s^{2\nu} + 1, \hat{k}(s) = \sum_{\mu = 0}^{m} \hat{k}_\mu s^{2\mu} \) using its roots in left half-plane (see third section (12) - (13) ).

6. Compute the estimate of time-delay \( \hat{\tau} \):

   a. Compute estimate of \( \hat{\tau} \) by substituting estimates of roots \( s^{[d]}_\nu \) \( \nu = 1 \rightarrow n \) into (16).

   b. Check the condition (17) with \( \hat{\tau} = \hat{\tau}^* \): if it is true then compute the estimate of time-delay \( \hat{\tau} \) by using (14) and (18); in other case the frequencies \( \omega_i (i = 1 \rightarrow l) \) must be chosen to satisfy (17) with \( \hat{\tau} = \hat{\tau}^* \), and goto step 1.

7. EXAMPLE

Let’s take the following transfer function of the plant (1). \( w(s) = \frac{1}{0.2s^3 + 1.24s^2 + 5.24s + 1} e^{-0.4s}. \) (21)

And choose the test signal:

\[
u(t) = 0.2 \sin(0.2001t) + 0.4 \sin(0.4002t) + +0.8 \sin(0.8004t) + 1.6 \sin(1.6008t).
\]

Example 1.

The external disturbance is meander \( f(t) = 0.5 \text{sign} [\sin(3t)] \); the measurement noise is the random noise \( \eta(t) = 0.1 \cdot \text{rand}. \)

The initial time moment is: \( t^*_F = 47.1 \text{ sec}. \)

The filtering time is: \( \mathcal{T}^* = 785 \text{ sec.} \)

Output of the plant is shown in figure 1.

Estimated plant obtained in the example 1 has a form

\[
\hat{w}(s) = \frac{0.9982}{0.2181s^3 + 1.1628s^2 + 5.1524s + 1} e^{-0.4144s}. \) (22)

It’s easy to see that coefficients of the estimated plant (22) are close to coefficients of the plant (21). Nyquist plots of the estimated plant (22) and the given plant (21) are shown on figure 2. It is easy to see that they are practically the same.
Output of the plant is shown in figure 3.

Estimated plant obtained in the example 2 has a form

\[
\hat{\nu}(s) = \frac{1.0008}{0.2381s^3 + 1.3897s^2 + 5.3230s + 1} e^{-0.3902s}. 
\] (23)

It’s easy to see that coefficients of the estimated plant (23) are close to coefficients of the plant (21). Nyquist plots of the estimated plant (23) and the given plant (21) are shown on figure 4. It is easy to see that they are practically the same.

8. CONCLUSION

In this paper the method of finite-frequency identification is extended for plants with time-delay in the presence of unknown-but-bounded disturbance and measurement noise. First plant coefficients are identified by applying Fourier filter. Then time-delay is obtained by formulas (16)-(18). It is shown the condition of convergence of identification is achieved when the external disturbance and the measurement noise are strongly FF-filtrable.

This development of the finite-frequency method provides new possibilities for identification of the real plants.

REFERENCES


Yamaguchi H., Nagumo K., and Abe N. Iterative design using imc for input time-delay systems with disturbance observer. pages 2430–2435, Milano, Italy, 2011.