ACCURATE ADAPTIVE CONTROL

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Abstract: An adaptation algorithm of controller coefficients for a linear plant with unknown coefficients in the presence of unknown-but-bounded disturbance is proposed. Control aim is the prescribed tolerance on the steady-state error of plant output. The algorithm makes use of a sufficiently small test signal that allows to identify the plant and closed-loop system. Controller is designed by procedure of H_∞ control modified in accordance with requirement of prescribed accuracy.

Keywords: Adaptive Control, Frequency Domain Identification, Unknown-but-bounded disturbance, Steady-state error

1. INTRODUCTION

The last two decades adaptive control in the presence of an unknown-but-bounded disturbance is being studied. A number of control algorithms have been developed based on the recurrrent target inequalities [1], least squares estimation with dead zone [2] and frequency approach [3].

In these methods, the control objective is described by a polynomial with prescribed pole placement, and this control is referred to as modal adaptive control [1].

For most cases, the control objective cannot be described by a polynomial, but rather contains requirements to steady-state error, maximum overshoot, settling time, etc.

The problem of accuracy of adaptive control (where the control objective is the accuracy of control for steady state) for the minimum-phase plant was solved in [4]. The frequency approach under unknown polyharmonic disturbance and minimum-phase plant was proposed in [5].

In this paper, the frequency approach is developed for the case of nonminimum-phase plant. It is based on the modal frequency adaptive control [3] and the technique [6] of accurate control of steady-state.

2. STATEMENT OF THE PROBLEM

Consider a completely controllable plant described by the following differential equation

$$\begin{gathered} y^{(n)} + d_{n-1}y^{(n-1)} + \cdots + d_n y = \\ k_m u^{(m)} + \cdots + k_0 u + m_0 f, \quad t \geq t_0 = 0, \end{gathered} \tag{1}$$

where $y(t)$ is a measured output, $u(t)$ is the input to be controlled, $y^{(i)}$, $u^{(j)}$ $(i=\overline{1,n}$, $j=\overline{1,m})$ are the derivatives of these functions, the coefficients d_i and k_j $\left(i=\overline{0,n-1}\right.$, $\left.j=\overline{0,m}\right)$ are some unknown numbers, n, $m<n$, and m_0 are known, $f(t)$ is the following polyharmonic function:

$$f(t) = \sum_{i=1}^{\infty} f_i \sin(\omega_i^f t + \phi_i^f), \tag{2}$$

where ω_i^f and ϕ_i^f, $i = 1, 2...$, are unknown frequencies and phases; the amplitudes f_i, $i = 1, 2...$, are unknown-but-bounded numbers satisfying the inequality

$$\sum_{i=1}^{\infty} |f_i| \leq f^*, \qquad (3)$$

where f^* is a given number.

The problem is to find a controller

$$\begin{aligned} g_n u^{(n)} + \cdots + g_0 u = \\ = r_{n-1} y^{(n-1)} + \cdots + r_0 y, \quad t \geq t_N \end{aligned} \qquad (4)$$

such that the plant output of system (1), (4) meet the following requirement:

$$y_{st} \leq y^*, \qquad (5)$$

where y_{st} is the steady-state deviation of the plan output which is defined as $y_{st} = \lim_{t\to\infty} \sup |y(t)|$ and y^* is a given positive number.

In order to solve the problem, the plant input is formed by the following controller with piecewise-constant coefficients

$$\begin{aligned} g_n^{[i]} u^{(n)} + \cdots + g_0^{[i]} u = \\ = r_{n-1}^{[i]} y^{(n-1)} + \cdots + r_0^{[i]} y + v^{[i]}, \\ t_{i-1} < t \leq t_i \quad i = \overline{1, N} \end{aligned} \qquad (6)$$

where i $\left(i = \overline{1, N}\right)$ is an adaptation interval number,

$$\begin{aligned} v^{[i]}(t) = \textstyle\sum_{k=1}^{\theta} \rho_k^{[i]} \sin \omega_k^{[i]} (t - t_{i-1}), \\ t_{i-1} \leq t < t_i \quad i = \overline{1, N} \end{aligned} \qquad (7)$$

are the test signals with the specified test frequencies $\omega_k^{[i]}$ and amplitudes $\rho_k^{[i]}$ $\left(k = \overline{1, \theta}, i = \overline{1, N}\right)$.

On some of adaptation intervals, particular for i=1,the differential equation (6) is algebraic equation $u = v^{[i]}$, $i \in [1, N]$. It means that the equation(6) has the coefficients $g_k^{[i]} = r_k^{[i]} = 0$ $\left(k = \overline{1, n}\right)$, $g_0^{[i]} = 1, r_0^{[i]} = 0$ In such cases the test signal (7) contains n harmonics ($\theta = n$) and in remaining cases $\theta = 2n$.

The frequencies of the disturbance and test signals must not coincide:

$$\omega_k^f \neq \omega_j^{[i]} \quad i = \overline{1, N} \quad j = \overline{1, \theta} \quad k = \overline{1, \infty} \qquad (8)$$

Since the disturbance frequencies are unknown it is necessarly to examine inequality (8) by experiment.To this effect the following functions are introduced

$$\begin{aligned} l_\alpha(\tau) = \frac{2}{\rho_k \tau} \int_0^\tau \bar{y}(t) \sin \omega_k^{[i]} t dt, \\ l_\beta(\tau) = \frac{2}{\rho_k \tau} \int_0^\tau \bar{y}(t) \cos \omega_k^{[i]} t dt, \\ k = \overline{1, \theta}, \quad i = \overline{1, N} \end{aligned} \qquad (9)$$

where $\bar{y}(t)$ is plant output when $v^{[i]}(t) = 0$ $\left(i = \overline{1, N}\right)$,
If

$$l_\alpha(\tau) \leq \epsilon_k^\alpha, \quad l_\beta(\tau) \leq \epsilon_k^\beta, \qquad (10)$$

where ϵ_k^α and ϵ_k^β $k = \overline{1, \theta}$ are sufficiently small given numbers, τ is sufficiently large number. If the conditions (10) hold then above mentioned frequencies do not coincide [3].

The amplitudes of test signal (7) have to meet the following conditions of small excitation [3]

$$\left| y^{[i]}(t) - \bar{y}^{[i]}(t) \right| \leq \epsilon_y, \quad i = \overline{1, N}, \qquad (11)$$

where $y^{[i]}(t)$ is the plant output on the ith interval of adaptation, $\bar{y}^{[i]}(t)$ is the same output for $v^{[i]} = 0$, ϵ_y is a given number. Requirements (11) mean that the test signal must not strongly change the "natural" output $\bar{y}^{[i]}(t)$.

In this paper ways of amplitudes tuning providing the conditions (11) as well as test frequencies tuning described in[9] is not considered and so upper index $[i]$ in notations is omited and it is assumed that these amplitudes and frequensies are specified.

The ending time of each adaptation interval t_i $\left(i = \overline{1, N}\right)$ is found in an adaptation prosses.In addition,these moments have to satisfy the following inequalities

$$t_i - t_{i-1} \geq t_{i-1} - t_{i-2} + t^*, \; i = 2, 3, \ldots, \qquad (12)$$

where t^* is a given positive integer to be suffitienly large. These inequalities is named conditions of wideness of adaptation intervals.

After ending of adaptation (in moment t_N) the controller described by equation (4) where

$$g_i = g_i^{[N]}, \quad r_i = r_i^{[N]} \quad i = \overline{0, n-1}. \qquad (13)$$

Problem 2.1 Find an adaptation algorithm for coefficients of controller (6) such that the system (1), (4) meet the demands (5) to steady-state accuracy.

3. PROBLEM SOLUTION FOR KNOWN PLANT COEFFICIENTS

Let the coefficients d_i and k_j $(i=\overline{0,n-1}, j=\overline{0,m})$ of plant (1) be known.

Equation (1) may be written in state space as

$$\dot{x} = Ax + b_1 f + b_2 u, \quad z = y = c_2 x \tag{14}$$

where

$$\begin{aligned} A &= \begin{pmatrix} 0 & 0 & \cdots & 0 & -d_0 \\ 1 & 0 & \cdots & 0 & -d_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -d_{n-1} \end{pmatrix}, \\ c_2 &= \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \end{bmatrix}, \\ b_1 &= \begin{bmatrix} m_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad b_2 = \begin{bmatrix} k_0 \\ k_1 \\ \vdots \\ k_m \end{bmatrix} \end{aligned} \tag{15}$$

z is the controlled variable coinciding with the measured output.

Consider a controller described by the following equations

$$\begin{aligned} u &= kx_c, \\ \dot{x}_c &= Ax_c + b_2 u + b_1 f_c x_c + k_c(y - c_2 x_c), \end{aligned} \tag{16}$$

where $x_c(t)$ is a state vector (its dimension is n) of the controller,

$$\begin{aligned} k = -b_2^T P, \quad f_c = \gamma^{-2} b_1^T P, \\ k_c = (I - \gamma^{-2} Y P)^{-1} Y c_2^T, \end{aligned} \tag{17}$$

P and Y are square non-negative definite matricies of dimension $n \times n$. They are solutions of the following algebraic Riccati equations

$$\begin{aligned} A^T P + PA - Pb_2 b_2^T P + \\ +\gamma^{-2} P b_1 b_1^T P = -c_2^T c_2 q_0 \end{aligned} \tag{18}$$

$$\begin{aligned} AY + YA^T - Y c_2^T c_2 Y + \\ +\gamma^{-2} Y c_2^T c_2 Y = b_1 b_1^T l_0 \end{aligned} \tag{19}$$

where γ is a number that is found such that together with non-negativeness of matricies P and Y the following condition hold

$$\lambda_{\max}(PY) < \gamma^2 \tag{20}$$

where $\lambda_{\max}(M)$ is a maximal eigenvalue of matrix M, q_0 and l_0 are some positive numbers ($q_0 = l_0 = 1$ in standard H_∞ optimal control [8]) that are determined such that the requirements (5) to stade-state errors hold.

Assertion 3.1 If the number q_0 meets the demands

$$q_0 \geq \frac{f^{*2}}{y^{*2}}, \tag{21}$$

then

$$y_{st}^2 \leq y^{*2} \gamma^{*2} l_0^{-1}, \tag{22}$$

where γ^* is a minimal value of γ. ■

Assertion proof is given in Appendix.

Assumption 3.1 There exsists number q_0 such that controller (16) with coeffitients (17) provides performance of requirements (5). ■

Coefficients of the searched equation (4) connect with the coefficients of equation (16) as

$$\begin{aligned} g(s) &= \det(Is - A_c), \\ r(s) &= k \operatorname{adj}(Is - A_c) k_c \end{aligned} \tag{23}$$

where adj is a simbol of the adjugate matrix,

$$A_c = A + b_2 k - k_c c_2 + b_1 f_c. \tag{24}$$

4. THE FIRST INTERVAL OF ADAPTATION

4.1 Plant identification.

If the plant (1) is asymptotically stable then solution of problem 2.1 is as following. Plant (1) is exited by test signal:

$$u(t) = v^{[1]}(t) = \sum_{k=1}^{n} \rho_k \sin \omega_k t, \tag{25}$$

and its output is applied to inputs of the following Fourier's filter

$$\begin{aligned} \hat{\alpha}_k &= \alpha_k(\delta) = \frac{2}{\rho_k \tau} \int_{t_F}^{t_F+\tau} y(t) \sin \omega_k (t - t_F)\, dt \\ \hat{\beta}_k &= \beta_k(\delta) = \frac{2}{\rho_k \tau} \int_{t_F}^{t_F+\tau} y(t) \cos \omega_k t(t - t_F)\, dt \\ & k = \overline{1,\theta}, \theta = n \ or\ \theta = 2n \end{aligned} \tag{26}$$

where τ is a filtering time, t_F is a filtering start time, τ and t_F is multiple some basic period $T = \frac{2\pi}{\omega_b}$, ($\tau = qT$, $t_F = \bar{q}T$, $q = 1, 2, \ldots$; $\bar{q}$ is a given interger), where ω_b is a basic frequency and test frequencies are multiple of number ω_b: $\omega_k = c_k \omega_k$ $\left(k = \overline{1,n}\right)$,$c_k$ $\left(k = \overline{1,n}\right)$ are positive integers,on this interval $\theta = n$, $\hat{\alpha}_k$ and $\hat{\beta}_k$, $(k = \overline{1,n})$ are the estimates of frequency domain parameters (FDPs)[3],which are a set $\alpha_k = Re\ w(j\omega_k)$,$\beta_k = Im\ w(j\omega_k)$, $k = \overline{1,n}$ where $w(s)$ is the plant transfer function.

The outputs of Fourier's filter are measured in time moments $\delta = qT$, $q = \bar{q} + 1$, $\bar{q} + 2, \ldots$.

The following frequency equations[3] are solved for these moments

$$\sum_{i=0}^{m}(j\omega_k)^i k_i(\delta) - [\alpha_k(\delta) + j\beta_k(\delta)] \cdot \sum_{i=0}^{\theta-1}(j\omega_k)^i d_i(\delta) = [\alpha_k(\delta) + j\beta_k(\delta)](j\omega_k)^\theta$$
$$k = \overline{1,\theta},\ \theta = n \text{ or } \theta = 2n \tag{27}$$

and estimates of plant coefficients $d_i(qT)$, $k_i(qT)$ $\left(i = \overline{0, n-1}\right)$, $q = \bar{q}+1,\ \bar{q}+2,\ \ldots$ are obtained.

In order to determine time of the first interval ending the following necessary conditions are examined

$$|d_i(qT) - d_i[(q-1)T]| \le \varepsilon_i^d,$$
$$|k_j(qT) - k_j[(q-1)T]| \le \varepsilon_j^k,$$
$$\imath = \overline{0, n-1}, j = \overline{0, m} \quad q = \bar{q}+2,\ \bar{q}+3, \ldots. \tag{28}$$

where ε_i^d and ε_j^k $(i = \overline{0, n-1}, j = \overline{0, m})$ are given numbers.

These inequalities are examined for each q till they hold for some $q = q_1$ and then $t_1 = q_1 T$.

4.2 Controller design .

Using the estimations $d_i^{[1]} = d_i(q_1T)$ and $k_j^{[1]} = k_j(q_1T)$, $(i = \overline{0, n-1}, j = \overline{0, m})$ the coefficients (17) of controller(16) is calculated. To this effect the Riccati equations (18) and (19), where q_0 is determined by inequality (21) and $d_i = d_i^{[1]}$, $k_j = k_j^{[1]}$, $i = \overline{0, n-1}$,$j = \overline{0, m}$,are solved. Then polynomials of controller(6)

$$g^{[2]}(s)u = r^{[2]}(s)y + v \tag{29}$$

for the second interval of adaptation are found by formulae(23)

Rewrite the system (1), (29) as

$$\varphi^{[2]}(s)y = k(s)v + g^{[2]}(s)f, \tag{30}$$

where

$$\varphi^{[2]}(s) = d(s)g^{[2]}(s) - k(s)r^{[2]}(s) \tag{31}$$

is characteristic polynomial of closed- loop system (1),(29)

Introduce an assumed polynomial of this system as

$$\psi^{[2]}(s) = d^{[1]}(s)g^{[2]}(s) - k^{[1]}(s)r^{[2]}(s) \tag{32}$$

It coincides with the polynomial $\varphi^{[2]}(s)$ when the identified and true plant polynomials are equal($d^{[1]}(s) = d(s)$, $k^{[1]}(s) = k(s)$) and therefore, differences of coefficients $\Delta_i = |\psi_i^{[2]} - \varphi_i^{[2]}|$, $(i = \overline{0, 2n})$ of polynomial $\psi^{[2]}(s)$ and $\varphi^{[2]}(s)$ characterizes identification accuracy.

5. THE SECOND INTERVAL OF ADAPTATION

5.1 Closed-loop system identification.

System (1), (29) is excited by test signal (7) (where $\theta = 2n$) and its output is applied to the following Fourier's filter (26), where $\theta = 2n$.Outputs of the filter are denoted as $\hat{\nu}_k$ and $\hat{\mu}_k$ where $\hat{\nu}_k$ and $\hat{\mu}_k$, $k = \overline{1, 2n}$, are estimates of the following closed-loop FDPs [3]:

$$\nu_k = Re\ w_{cl}(j\omega_k), \quad \mu_k = Im\ w_{cl}(j\omega_k), k = \overline{1, 2n}$$

Here $w_{cl}(s) = \frac{k(s)}{\varphi^{[2]}(s)}$ is a transfer function of the closed-loop system.

It is obviously that this transfer function connects with the transfer function of plant as

$$w_{cl}(s) = \frac{w(s)w_e^{[2]}(s)}{1 - w(s)w_c^{[2]}(s)} \tag{33}$$

where

$$w_c^{[2]}(s) = \frac{r^{[2]}(s)}{g^{[2]}(s)}, \quad w_e^{[2]}(s) = \frac{1}{g^{[2]}(s)}.$$

Outputs this Fourier's filter allow to find coefficients estimates of characteristic polynomial of system (1),(29) by solution of the frequency equations (27), in which $\theta = 2n$, $\delta = q_1 + 1, q_2 + 2, ..., q_2^{(1)}$ and $q_2^{(1)}$ is determined from condition (12) that may be rewritten as

$$q_i - q_{i-1} \ge q_{i-1} - q_{i-2} + k^*, (i = 1, 2, ...) \tag{34}$$

where $k^* = [\frac{t}{T}]$ is a interge part of number $\frac{t}{T}$.

Necessary conditions of the closed-loop identification convergence are

$$\begin{gathered} |\varphi_i^{[2]}(qT) - \varphi_i^{[2]}[(q-1)T]| \le \varepsilon_i^\varphi, \\ |k_j(qT) - k_j[(q-1)T]| \le \varepsilon_j^k, \\ i = \overline{0, 2n}, \qquad j = \overline{0, m} \end{gathered} \tag{35}$$

where ε_i^φ $(i = \overline{0, 2n})$ are given positive numbers.

If for $q = q_2^{(1)}$ inequalities(35) are violated then the identification is continued till they hold at a moment $q_2^{(2)}T$.

5.2 Interval duration.

Duration of the second interval is determined by the following inequalities

$$|\psi_i^{[2]} - \varphi_i^{[2]}(\delta)| \leq \varepsilon_i^{\psi}, (i = \overline{0, 2n}), \qquad (36)$$

where $\delta = q_2^{(1)}T$ or $\delta = q_2^{(2)}T$ and ε_i^{ψ}, $(i = \overline{0, 2n})$ are given numbers. During interval $t_2 - t_1$ the following conditions

$$|y(t)| \leq y^* - \varepsilon_y \qquad (37)$$

are examine.It means fulfilling requirement to accuracy (5).

Now it is possible four cases:(a),(b),(c) and (d). Consider each of them.

*Case (a).*If at a moment $t_2 = q_2^{(2)}T$ the condition (36) hold and the requirement (37) to accuracy is satisfied for some $t^*(t_1 \leq t^* \leq q_2^{(2)}T)$ such that the difference $t^* - t_1$ is sufficiently large, then the second interval is ended and $N = 2$, $t_2 = t_2^{(a)} = t_N$.The searched polynomials of controller (4) are $g(s) = g^{[2]}(s)$, $r(s) = r^{[2]}(s)$.

Case (b). If condition (36) hold at the moment $t_2 = q_2^{(2)}T$ but the moment t^*,for which the requirement (37) satisfy, does not exist then the second interval is ended and $t_2 = t_2^{(b)} = q_2^{(2)}T$.

Case (c). Let condition (36) be violated at the moment $t_2 = q_2^{(2)}T$.It means,in particular, that identification accuracy, obtained as a result of the first interval,is not sufficiently and so the plant identification is continued. To this effect, the estimates of the plant FDPs $\alpha_k(\delta)$ and $\beta_k(\delta)$ $(k = \overline{1, n})$ are calculated by the following formulae

$$\alpha_k(\delta) + j\beta_k(\delta) = \\ = \frac{\nu_k(\delta) + j\mu_k(\delta)}{[\nu_k(\delta) + j\mu_k(\delta)]\, w_c^{[2]}(j\omega_k) + w_e^{[2]}(j\omega_k)}, \\ k = \overline{1, n}, \qquad (38)$$

where $\delta = qT$, $q = q_2^{(2)} + 1, q_2^{(2)} + 2, ...,$ Expression(38) follows from equality (33). Using new estimates of the plant FDPs the frequency equations (27) are solved and necessary conditions(28) are examined for $q_2^{(2)} + 1, q_2^{(2)} + 2, ...,$till they hold for some $q = q_2^{(3)}$,which has to satisfy the condition: $q_2^{(3)} - q_2^{(2)} > q_1 + k^*$, and then $t_2 = t_2^{(c)} = q_2^{(3)}T$

Repeating steps of subsection 4.2 for $d_i^{[2]} = d_i(q_2^{(3)}T)$ and $k_j^{[2]} = k_j(q_2^{(3)}T)$, $(i = \overline{0, n-1}, j = \overline{0, m})$ the polynomials of the following controller

$$g^{[3]}(s)u = r^{[3]}(s)y + v \qquad (39)$$

for the third interval of adaptation and the assumed polynomial of system (1),(39):

$$\psi^{[3]}(s) = d^{[2]}(s)g^{[3]}(s) - k^{[2]}(s)r^{[3]}(s) \quad (40)$$

are found.

Case (d) If at a moment $t_2^{(d)}$ the plant output $y(t_2^{(d)}) = y^{**}$, where y^{**} is a maximal allowable output (for example,when $|y(t)| > |y^{**}|$ the plant may be not described by the equation (1)), then the second interval is ended and $t_2 = t_2^{(d)}$. This case arises,for instans, when system (1),(29) is unstable.

6. ADAPTATION CONVERGENCE

6.1 The third interval.

A contents of the third interval depends on the cases (b), (c) and (d) that have arisen on the second interval.Let us continue considering each of them.

Case(b). A cause of this case is relative large values of adaptation algorithm parameters: $\varepsilon_i^d, \varepsilon_j^k$, $i = \overline{0, n-1}$, $j = \overline{0, m}$, $\varepsilon_i^{\varphi}, \varepsilon_j^{\psi}$ $i, j = \overline{0, 2n}$. In connection with it these parameters is decreased,for example they are divided by two,and the operations of the second interval is repeated.

Case (c). The operations of the second interval with controller (39) and the assumed polynnomial (40) are carried out.

Case(d). In this case the controller (29) is switched off and plant (1) is exited by test signal (6) under $\theta = n$ and the operations of the first interval are repeated,however, duration of this interval is more than the first one: $q_3 - q_2 \geq q_1 + k^*$

6.2 Adaptation convergence.

If the frequencies of the disturbance and the test signal do not coincide (condition (8) hold) then functions (9) tend to zero: $\lim_{\tau\to\infty} l_k^{\alpha}(\tau) = \lim_{\tau\to\infty} l_k^{\beta}(\tau) = 0$, $k = \overline{1, \theta}$ and therefore identification errors $\Delta d_i(\delta) = d_i - d_i(\delta)$ $\Delta k_j(\delta) = k_j - k_j(\delta)$ $(i = \overline{0, n-1}, j = \overline{0, m})$ tend to zero as well.It means that it exist a number δ^* such that $|\Delta d_i(\delta)| \leq \bar{\varepsilon}_i^d$, $|\Delta k_j(\delta)| \leq \bar{\varepsilon}_j^k$ $(i = \overline{0, n-1}$, $j = \overline{0, m})$, $\delta \geq \delta^*$, where $\bar{\varepsilon}_i^d$ and $\bar{\varepsilon}_j^k$ $(i = \overline{0, n-1}, j = \overline{0, m})$ are any given small numbers.

Problem is to achive the filtering time δ^*.This value is achived due to the conditions (12) of adaptation intervals widenness and decreasing adaptation algorithm parameters.

Identification convergence allows to find a controller for which the accuracy requirement (5) is fulfilled.

So,the following assertion is obviously.

Assertion 6.1 If the test frequencies satisfy the condition (8) then adaptation process converges to a controller of view (4) which provides the achivement of aim (5).

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8. APPENDIX

8.1 Proof of assertion 3.1.

Denote $t_{yf}(s)$ and $t_{uf}(s)$ the transfer functions of system (14), (16):$t_{yf}(s)$ is a transfer function from f to y, $t_{uf}(s)$ does from f to u.

Output of this system in the presence of disturbance (2) when the time t tends to infinity is

$$y(t) = \sum_{i=1}^{\infty} a(\omega_i^f)\sin(\omega_i^f t + \kappa_i) \tag{41}$$

where $a(\omega_i^f)$ and κ_i $(i = 1, 2, ...)$ are amplitudes of forced oscillations and their phases.

This expression result in the following estimate of steady-state output

$$y_{st} \leq \sum_{i=1}^{\infty} |a(\omega_i^f)| \tag{42}$$

It is obviously that

$$|a(\omega_i^f)| = |t_{yf}(j\omega_i^f)||f_i| \quad (i = 1, 2, ...) \tag{43}$$

Proof the assertion based on theorem [7] which for $l_0 \neq 1$ may be formulated as: if coefficients of controller (16) is found by formulae (17)-(19) and condition (20) hold then the transfer function of system (14),(16) satisfies the followig inequality

$$q_0|t_{yf}(j\omega)|^2 + |t_{uf}(j\omega)|^2 \leq \gamma^2 l_0^{-1}, \quad 0 \leq \omega \leq \infty \tag{44}$$

Making use of this inequality the expression (43) may be written as

$$|a(\omega_i^f)| \leq \frac{\gamma|f_i|}{\sqrt{q_0 l_0}} \quad (i = 1, 2, ...) \tag{45}$$

Adding these inequalities and taking into account the boundary (3) it results in

$$\sum_{i=1}^{\infty} |a(\omega_i^f)| \leq \frac{\gamma f^*}{\sqrt{q_0 l_0}} \tag{46}$$

and therefore

$$y_{st} \leq \frac{\gamma f^*}{\sqrt{q_0 l_0}} \tag{47}$$

Estimates (22) follows from this inequality under condition (21).