Proceedings of the 8th IFAC Symposium on Robust Control
Design, Bratislava, Slovak Republic, July 8-11, 2015

Design of H_∞ Controllers under Parametric Uncertainty and Power-Bounded External Disturbances

Vladimir N. Chestnov

V.A.Trapeznikov Institute of Control Science, Russian Academy of Sciences, Moscow, Russia, 117997
(Tel: +7495-334-7641; e-mail: vnchest@rambler.ru)

Abstract: We consider robust stabilization of linear MIMO systems, whose physical parameters are allowed to deviate from the nominal ones within known bounds, and the control plant is subjected to unknown power-bounded polyharmonic external disturbances (with unknown amplitudes and frequencies). The problem is to design a robustly stabilizing controller such that the prespecified errors for the controlled variables in steady state are guaranteed. The solution is based on the "loop-breaking technique" of the plant-controller system with respect to the physical parameters, e.g. Chestnov (1999); it reduces to the standard H_∞-optimization procedure by properly choosing the weighting matrix at the controlled variables. This approach is implemented numerically in the MATLAB-based Robust Control Toolbox (RCT).

Key words: linear multivariable systems, parametric uncertainty, bounded external disturbances, output feedback controller, H_∞-optimization, linear matrix inequalities.

1. INTRODUCTION

Robust stabilization of systems subjected to deviations of the parameters from their nominal values is the subject of numerous papers and books; e.g., see Ackermann (1993), Barmich (1994), Bhattacharyya et al. (1995), Zhou, et al. (1995), Zhou, et al. (1998). At the same time, as a rule, the majority of the papers consider the model parameters as the entries of the matrices in the state space equations or the coefficients of the transfer matrix of the plant. Generally speaking, these cannot be associated with the physical parameters of the plant, since both state-space equations and transfer matrix are secondary description tools for dynamic systems. Indeed, they are derived by converting the original equations of the dynamic system formulated from the fundamental laws of physics (mechanics, electro-dynamics, etc.). In this work, we deal with the parameters of such an intrinsic description, which have a transparent physical meaning, such as mass, moment of inertia, ohmic resistance, capacitance, inductance, etc. Moreover, as a rule, such a conversion of the original physical description leads to ``mixing'' and ``duplication'' of the varying parameters and hence, to a considerable complication of the problem and essentially conservative end-results.

In practice, real dynamic systems are affected by unknown external disturbances; in the mathematical control theory these disturbances are usually assumed to be bounded in certain norm, (e.g., see Chestnov (2011), Skogestad et al. (2007), Zhou et al. (1995), Zhou et al. (1998).

The approach developed here leans on the representation of dynamic systems in the so-called canonical (W, Λ, K)-form, (see Chestnov (1985), Chestnov (1995), Chestnov (1999)) such that, in the plant, the physical parameters (subjected to deviations from their nominal values) make up internal fictitious feedback loops in the form of the diagonal Λ matrix.

Unknown external disturbances are taken in the form of polyharmonic signals (with unknown amplitudes and frequencies), which are assumed to be power-bounded (i.e., for every coordinate of the disturbance, the sum of the squared amplitudes of each polyharmonic component is bounded). Similarly to Chestnov (2011), for the dynamic system we introduce the notion of the radius of the steady state with respect to the controlled variables. On top of robust stability, the desired controller must guarantee the specified (or, the minimal possible) value of the radius.

We show that such a problem reduces to the standard problem of rejection of exogenous disturbances in the framework of H_∞-approach (see Doyle et al. (1989)) by appropriately choosing the weighting matrix at the controlled variables of the plant.

This approach is implemented numerically as a code in the MATLAB-based Robust Control Toolbox (see Balas et al. (2010)). The idea of robust design using the (W, Λ, K)-representation was first proposed in Chestnov (1999), while the account for the accuracy is similar to the one in Aleksandrov, Chestnov (1998, a,b), Chestnov (2011). The design methodology is exemplified through the benchmark problem borrowed from in Haddad et al. (1993) and Farag et al. (2002).

2. STATEMENT OF THE PROBLEM

We consider a control plant described by the following equations in the physical variables:

$$\begin{cases} L_1(p)z_0(t) = L_2(p)u(t) + L_3(p)f(t) \\ y(t) = Nz_0(t) \end{cases} \qquad (1)$$

where z_0 is the l-dimensional vector of the physical variables of the plant (velocity, acceleration, current, displacement, angle of rotation, etc.); u is the m-dimensional control input; y is the m_2-dimensional vector of observable (controllable) variables of the plant, f is the m_3-dimensional vector of external disturbances; N is a known numerical matrix of dimension $(m_2 \times l)$; $L_1(p)$, $L_2(p)$, $L_3(p)$ are polynomial matrices of dimensions $(l \times l)$, $(l \times m)$, $(l \times m_3)$, respectively, in the differentiation operator $p = d/dt$:

$$L_1(p) = \sum_{i=0}^{\alpha_1} L_1^{(i)} p^i, \qquad L_2(p) = \sum_{j=0}^{\alpha_2} L_2^{(j)} p^j,$$

$$L_3(p) = \sum_{k=0}^{\alpha_3} L_3^{(k)} p^k, \qquad \alpha_2, \alpha_3 < \alpha_1$$

where $L_1^{(i)}, L_2^{(j)}, L_3^{(k)}$ are known real matrices of compatible dimensions.

In what follows, it is assumed that plant (1) is stabilizable and detectable, and its equations correspond to the original, ``least transformed'' description obtained from the fundamental laws of physics. In the sequel, the entries of the matrices will be referred to as the physical parameters of the plant. It is also assumed that n of the parameters entering (1) have nominal values $\lambda_1, \lambda_2, \ldots, \lambda_n$ and are allowed to vary in the given intervals:

$$\lambda_i + \Delta\lambda_i \in (\lambda_i^{\min}, \lambda_i^{\max}), \quad i = \overline{1, n}, \tag{2}$$

where $\Delta\lambda_i$ is the deviation of the i-th parameter from its nominal value.

Matrix elements $L_3(p)$ leave unaffected the stability of the closed-loop system, and therefore the deviations from the calculated not further considered here.

The coordinates of the vector external disturbance f are represented by bounded polyharmonic functions of the form

$$f_i(t) = \sum_{k=1}^{p_0} f_{ik} \sin(\omega_k t + \psi_{ik}), \quad i = \overline{1, m_3}. \tag{3}$$

Here, the amplitudes f_{ik}, the initial phases ψ_{ik} ($i = \overline{1, m_3}$, $k = \overline{1, p_0}$), and the frequencies ω_k ($k = \overline{1, p_0}$) of the harmonics are not known; however, the amplitudes satisfy the bounds

$$\sum_{k=1}^{p_0} f_{ik}^2 \le w_i^{*2}, \qquad i = \overline{1, m_3}, \tag{4}$$

where p_0 is a known number of harmonics, and the numbers w_i^* ($i = \overline{1, m_3}$) are given.

We define the steady-state errors with respect to the controlled variables by the following relation:

$$y_{i,st} = \lim_{t\to\infty} \sup |y_i(t)|, \quad i = \overline{1, m_2}. \tag{5}$$

Require that the output feedback controller provides the following conditions:

$$y_{i,st} \le y_i^*, \quad i = \overline{1, m_2}, \tag{6}$$

where $y_i^* > 0$, $i = \overline{1, m_2}$, are given numbers.

Clearly, there might exist no such controllers (see Aleksandrov et al. (1998, a)). Introduce the notion of the steady-state radius for the closed-loop system with respect to the controlled variables (e.g., see Aleksandrov, Chestnov (1998, b), Chestnov (2011)):

$$r_{st}^2 = \sum_{i=1}^{m_2} \left(\frac{y_{i,st}}{y_i^*} \right)^2, \tag{7}$$

which will be limited.

Problem *1*: Synthesize a stabilizing output feedback controller

$$u(t) = K(p) y(t), \tag{8}$$

with the proper transfer matrix $K(p)$ such that the following holds:

(i) for given finite deviations of the parameters $\lambda_1, \lambda_2, \ldots, \lambda_n$ from the nominal (2), the closed-loop system retains the asymptotic stability;

(ii) the steady-state radius for the controlled variables satisfy

$$r_{st}^2 \le \gamma^2, \tag{9}$$

where γ is a given (or minimal possible) number.

Obviously, for the problem to possess a solution, the assumption should be adopted on the retention of stabilizability and detectability of plant (1) under variations of the parameters within intervals (2).

To solve the problem, we follow the ``loop-breaking'' technique with respect to the varying parameters (e.g., see Chestnov (1999)) and represent the closed-loop equations (1), (8) in the diagonal canonical (W, Λ, K) – form with account for external disturbance (3).

3. THE CANONICAL (W, Λ, K)–FORM

The canonical (W, Λ, K)-representation of the closed-loop system with external disturbances has the form (see Chestnov (1985), Chestnov (1995), Chestnov (1999)):

$$\begin{aligned} \tilde{y} &= W_{11}\tilde{u} + W_{12}u + W_{13}f, \quad \tilde{u} = \Lambda\tilde{y} \\ y &= W_{21}\tilde{u} + W_{22}u + W_{23}f, \quad u = Ky \end{aligned} \tag{10}$$

where $W_{ij}(s)$ $(i=1,2,\ j=1...3)$ are known transfer matrices which do not contain the varying parameters; u, y are, respectively, the physical input and output of plant (1); $\tilde{u}$, $\tilde{y}$ are n - dimensional fictitious input and output of the plant; $\Lambda = \mathrm{diag}[\lambda_1, \lambda_2, \ldots, \lambda_n]$ is the diagonal matrix of the parameters subjected to deviations around the nominal; K is the desired transfer matrix of controller (8).

The block-diagram associated with representation (10) is depicted in Figure 1.

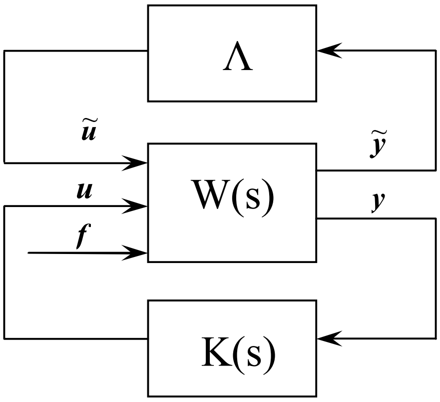

Fig. 1: Block-diagram of the system (10)

Theorem *1*: The closed-loop equations (1), (8) can always be represented in the equivalent (W,Λ,K) – form (10).

The proof is of constructive nature (see Chestnov (1995)).

Notice that the first proposed (W, Λ, K)-form (see Chestnov (1985)) significantly differs from proposed (M, Δ)-configuration (see Safonov (1981), Safonov (1982), Doyle (1982), Doyle (1983)).

The deviations of the parameters are taken out in the (M, Δ)-configuration. The main difference the (W, Λ, K)-form is the parameters are taken out of the plant in the (W, Λ, K). It allows using not only small-gain theorem but the Nyquist criteria and its generalizations.

4. APPROACH TO SOLVE THE PROBLEM

At first we consider the approach to the fulfillment of the condition (i), however, the accuracy requirements (9) omitted.

The transfer matrix of open-loop system (10) of variable parameters λ_i $(i=\overline{1,n})$, when breaking is the vector $\tilde{u}$, can be written as:

$$W^{\tilde{u}} = \Lambda[-W_{11} - W_{12}K(I - W_{22}K)^{-1}W_{21}] \qquad (11)$$

As seen from (11) feature of the transfer matrix is that the variable parameters are diagonal matrix of the gains in it.

If this frequency transfer matrix satisfies the inequality

$$[I + W^{\tilde{u}}(-j\omega)]^T [I + W^{\tilde{u}}(j\omega)] > r^2 I, \quad \omega \in [0,\infty], \qquad (12)$$

where I is the identity matrix of appropriate size; r is radius of stability margins $(0 < r \le 1)$.

Then the following sufficient estimators for intervals of possible values of the parameters is the case (see Chestnov (1985), Chestnov (1995), Chestnov (1999)):

$$\min\left\{\frac{\lambda_i}{1+r}, \frac{\lambda_i}{1-r}\right\} \le \lambda_i + \Delta\lambda_i \le \max\left\{\frac{\lambda_i}{1+r}, \frac{\lambda_i}{1-r}\right\}, \ i = \overline{1,n} \qquad (13)$$

which guarantee the robust stability of the system (1), (8).

In the single variable case (n = 1), fulfillment of the inequality (12) implies that the Nyquist diagram $W^{\tilde{u}}(j\omega)$ don't crossing the circle of radius r centered at the critical point (-1, j0) in the hodograph plane.

In the multivariable case (n > 1) frequency condition (12) has the following physical interpretation: the gains you can vary from the nominal unit value in the range (1/(1 + r), 1/(1 - r)), independently of the other gains, without loss of the stability for each of the fictitious inputs of the plant (see Lehtomaki et al.(1981))$\tilde{u}_i$, $(i=\overline{1,n})$. This implies the boundaries of robust stability (13).

Thus, the solution the first part (i) of Problem 1 is reduced to such a construction of the matrix K of the controller (8), that the number r takes required value or maximized. This problem was solved in Chestnov (1999).

5. REDUCING THE PROBLEM TO THE STANDARD H_∞-PROBLEM

We show that the problem of ensuring the given radius of stability margins r, or its maximization ($0 < r \le 1$) as well as the guaranteeing of the prespecified steady-state radius, is reduced to a standard problem of H_∞ optimization.

Here, in contrast to Chestnov (1999) the effect of the disturbances f is taken into account.

Consider the closed-loop system shown in Fig. 2.

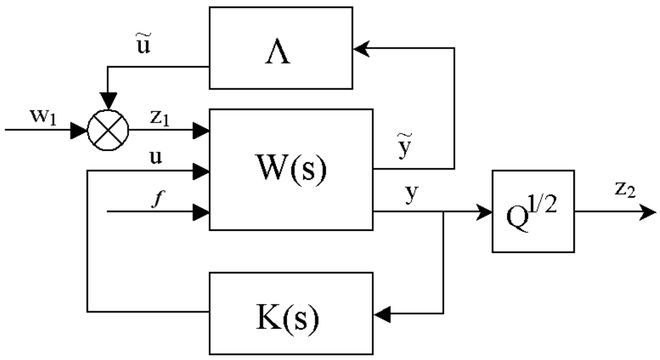

Fig. 2 – Block-diagram of the closed-loop system

We introduce extented vector of the disturbances and extented vector of the controlled output as components

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \quad w = \begin{bmatrix} w_1 \\ f \end{bmatrix}, \ (Q = diag[q_1, q_2, \ldots q_{m_2}], q_i > 0),$$

and consider the reduction of the Problem 1 to the standard problem of H_∞ optimization.

The system on Fig. 2 is described by the following equations:

$$\begin{aligned} &\tilde{y} = W_{11}z_1 + W_{12}u + W_{13}f, \ \tilde{u} = \Lambda\tilde{y}, \ z_1 = \tilde{u} + w_1, \\ &y = W_{21}z_1 + W_{22}u + W_{23}f, \ u = Ky, \ z_2 = Q^{1/2}y. \end{aligned} \qquad (14)$$

The transfer matrix of the closed-loop system, connecting the vector z and w, denoted as T_{zw}:

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = T_{zw} \times w = \begin{pmatrix} T_{z_1 w_1} & T_{z_1 f} \\ Q^{1/2}T_{yw_1} & Q^{1/2}T_{yf} \end{pmatrix} \times \begin{bmatrix} w_1 \\ f \end{bmatrix}. \qquad (15)$$

Let the sought controller $K(s)$ solves the H_∞-problem:

$$\|T_{zw}\|_\infty \le \gamma . \quad (16)$$

Then each of the blocks of the matrix satisfies the same condition, in particular

$$\|T_{z_1w_1}\|_\infty \le \gamma, \qquad \|Q^{1/2}T_{yf}\|_\infty \le \gamma . \quad (17)$$

The first of the inequalities (17) in the frequency form is equivalent to (12), where $r=1/\gamma$ (see Chestnov (1995)).

The second inequality (17) also can be represented in the equivalent frequency form

$$T_{yf}^T(-j\omega)QT_{yf}(j\omega) \le \gamma^2 I, \quad \omega \in [0,\infty] . \quad (18)$$

Given the diagonal structure of the matrix Q and the lemma of the steady-state values (see Aleksandrov et al. (1998, a), Chestnov (2011)); we obtain the following inequality for the steady-state errors of controlled variables

$$\sum_{i=1}^{m_2} q_i y_{i,st}^2 \le p_0 \gamma^2 \|w^*\|^2 ,$$

where $\|w^*\|$ is the Euclidean norm of the vector w^* with the components of the right part of (4).

Then, choosing the elements of the weight matrix Q from equalities

$$q_i = \frac{p_0\|w^*\|^2}{y_i^{*2}}, \qquad i = \overline{1, m_2} \quad (19)$$

will come to fulfillment of the condition (9).

Thus, solving the H_∞-problem (16), we achieve a resolution of the problem 1.

The problem of finding the proper transfer matrix of the controller $K(s)$, which ensures that the inequalities (9) and (12) are satisfied, can be rewritten as the following H_∞ control problems .

Problem *1.1* (Optimal H_∞ control**)**: Find proper transfer matrix of the controller (8), which would ensure the validity of (9) and (12) with the least possible $\gamma = \gamma_0$.

Problem *1.2* (Suboptimal H_∞ control**)**: Given the number $\gamma > \gamma_0$ find the proper transfer matrix controller (8) such that the inequalities (9) and (12) are satisfied.

If the problems 1.1 and 1.2 are solved, then the radius of stability margins $r = \gamma^{-1}$, and sufficient estimation at the intervals of possible values of the parameters, which guarantee the robust stability of the system (1), (8), follows from (13).

Let's give the equations (14) we write in the standard form, adopted in the H_∞ theory:

$$z = G_{11}w + G_{12}u, \quad y = G_{21}w + G_{22}u, \quad u = Ky. \quad (20)$$

The transfer matrix $G_{ij}(s)$ we obtain from (14).

The transfer matrix $G_{ij}(s)$ $(i, j = 1,2)$ of the generalized plant are associated with the transfer matrix equations (14) by equalities:

$$G_{11} = \begin{bmatrix} (I-\Lambda W_{11})^{-1} & (I-\Lambda W_{11})^{-1}\Lambda W_{13} \\ Q^{1/2}W_{21}(I-\Lambda W_{11})^{-1} & Q^{1/2}W_{21}(I-\Lambda W_{11})^{-1}\Lambda W_{13} + Q^{1/2}W_{23} \end{bmatrix}$$
$$G_{12} = \begin{bmatrix} (I-\Lambda W_{11})^{-1}\Lambda W_{12} \\ Q^{1/2}W_{21}(I-\Lambda W_{11})^{-1}\Lambda W_{12} + Q^{1/2}W_{22} \end{bmatrix}$$
$$G_{21} = \left[W_{21}(I-\Lambda W_{11})^{-1} \quad W_{21}(I-\Lambda W_{11})^{-1}\Lambda W_{13} + W_{23}\right] \quad (21)$$
$$G_{22} = \left[W_{21}(I-\Lambda W_{11})^{-1}\Lambda W_{12} + W_{22}\right]$$

These transfer matrices are derived of (14) by the simple substitution.

6. DESIGN PROCESS

Represent the synthesis procedure in the following sequence.

1. Bring the system equation (1), (8) to (10), where Λ - a diagonal matrix, which includes the selected designer ratings of the physical parameters of the system, subject to deviations from the nominal values.

2. Write (14) in the standard form, (20) adopted in the theory H_∞ control, taking into account (21) and brings it to the state-space equations.

3. To solve the problem of optimal or suboptimal H_∞ control (16) with (19) and to find the transfer matrix controller (8) $K(s)$.

4. Find a guaranteed tolerance limits on the parameters of the plant by (13) and steady-state errors with respect to the controlled variables from (9).

5. Compare found in Item 4 with the specified tolerance limits (2) and (6).

7. EXAMPLE

We illustrated the proposed method of synthesis by the example of a two-mass system (two carriages connected by a spring), which was used as a benchmark problem (e.g., see Haddad et al. (1993)) for many methods of synthesis of robust systems.

The equations of the plant are as follows:

$$\dot{x}_1 = x_3,\ \dot{x}_2 = x_4,\ \dot{x}_3 = -qx_1 + qx_2 + u + f,\ \dot{x}_4 = qx_1 - qx_2,$$

where q is the interval parameter (nominal q=0.8) (rigidity of spring), u is the control, and $y = x_2$ is the measured variable.

According to the synthesis procedure of Sec. 6, we reduce the equations of the plant to the canonical form (10) and introduce to this end the notation

$$\tilde{u} = q(x_2 - x_1),\ \tilde{y} = x_2 - x_1,\ \lambda = q.$$

After simple transformations, we rewrite the equations of the plant in the form:

$$\tilde{y} = -\frac{2}{s^2}\tilde{u} - \frac{1}{s^2}(u+f),\ y = -\frac{1}{s^2}\tilde{u},\ \tilde{u} = \lambda\tilde{y},\ u = K(s)y,$$
where $K(s)$ is the unknown transfer function (since u and y are scalars) of the controller, and the diagonal matrix Λ is just a scalar λ coinciding with the parameter q. One can easily see last form that the elements of the transfer matrix $W(s)$ are scalar transfer functions

$$W_{11} = -\frac{2}{s^2},\ W_{12} = W_{13} = -\frac{1}{s^2},\ W_{21} = -\frac{1}{s^2},\ W_{22} = W_{23} = 0.$$

We obtain this form, by means of (21), the elements of the transfer matrix $G(s)$ of the generalized plant of the standard H_∞-configuration (20), in particular

$$G_{22}(s) = \frac{q}{s^2(s^2+2q)}.$$

We note that $G_{22}(s)$ is the transfer function of the controlled plant $(u \to y)$; its characteristic polynomial $d(s) = s^2(s^2+2q)$ has a double zero root and two purely imaginary complex-conjugate roots.

To solve the problem of synthesis we reduce the equations of the generalized plant (20) to equations in the state space

$$\dot{x} = Ax + B_1 w + B_2 u,\ z = C_1 x + D_{11} w + D_{12} u,\ y = C_2 x + D_{21} w + D_{22} u$$
with regard for transfer matrix $G(s)$.

This reduction results in

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -q & q & 0 & 0 \\ q & -q & 0 & 0 \end{bmatrix},\quad B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ -1 & 0 \end{bmatrix},\quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -q \quad q \quad 0 \quad 0 \\ Q^{1/2}\cdot(0 \quad 1 \quad 0 \quad 0) \end{bmatrix},\qquad C_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix},$$

$$D_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\qquad D_{12} = D_{21} = D_{22} = 0\cdot$$

Since $D_{12} = D_{21} = 0$ we deal with a singular problem. The approach of Doyle J.C. et al. (1989) is unsuitable for the solution of problem (16). For this purpose the method of linear matrix inequalities (LMI) in the package Balas G.J. et al. (2010) is efficient.

Let's compare controller obtained by the method unaware of external disturbances (see Chestnov (1999)) and controller obtained by the suggested here approach.

The transfer function of the controller obtained with the method of Chestnov (1999) is

$$K_1(s) = \frac{-1225s^3 + 8016s^2 - 112.1\cdot 10^8 s - 5.697}{s^4 + 17.12s^3 + 195s^2 + 1178s + 4470}$$

and suboptimal γ is 1.1701; $r = \gamma^{-1}$=0.8546.

The transfer function of the controller constructed with the method proposed here is (Q=1, y*=1, w*=1, p_0=1)

$$K_2(s) = \frac{-3.1\times10^8 s^3 - 4.2\times10^8 s^2 - 4.6\times10^8 s - 1.6\times10^8}{s^4 + 193.7s^3 + 2.2\times10^4 s^2 + 1.4\times10^6 s + 6.1\times10^7}$$
with suboptimal γ is 1.1306; $r = \gamma^{-1}$=0.8845.

Bode magnitude responses (Fig.3 and Fig.4) show that the worst external disturbance is step in both cases.

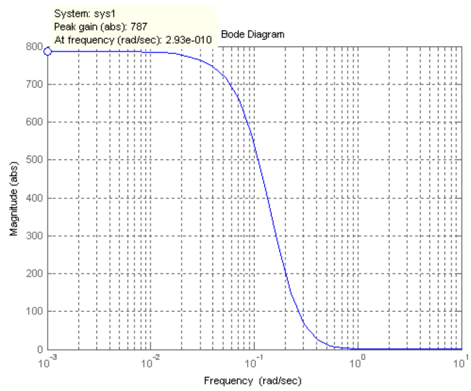

Fig. 3 The bode magnitude response with the controller obtained with the method of Chestnov (1999)

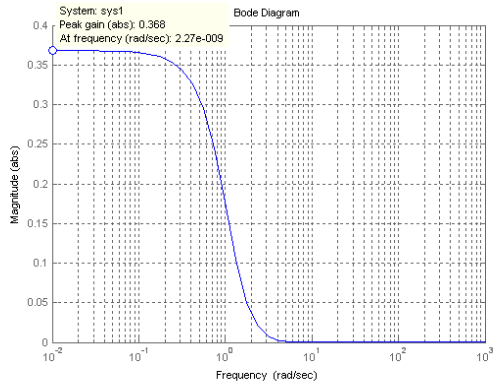

Fig. 4 The bode magnitude response with the controller constructed with the method proposed here

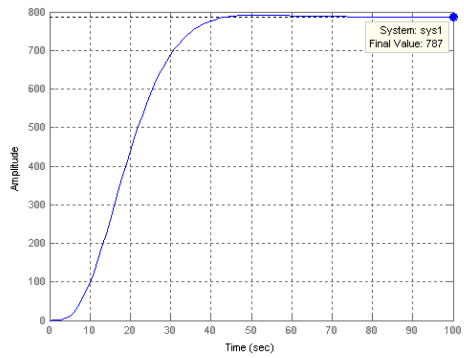

Fig. 5 The step response with the controller obtained with the method of Chestnov (1999)

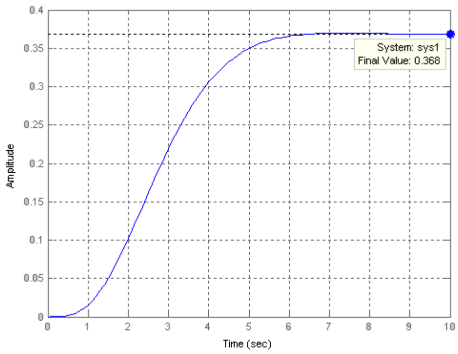

Fig. 6 The step response with the controller constructed with the method proposed here

Note that for the controller $K_1(s)$ obtained with the method of Chestnov (1999) guaranteed bounds on the parameter q are: $0.4314<q<5.5022$. For the controller constructed with the method proposed here $K_2(s)$ guaranteed bounds on the parameter q (found from (13)) are: $0.4245<q<6.9264$, which is wider. Both tolerances are better than that of known (e.g. see Haddad et al. (1993) and Farag et al. (2002)). The steady state error for the controller $K_2(s)$ (see Fig. 6) is significally better than the one for the controller $K_1(s)$ (see Fig. 5).

8. CONCLUSIONS

We note the advantages of these results.

1. Consideration is given to the deviations of the physical parameters from the nominal ones.

2. The optimization criterion (radius of the stability margin and steady-state radius) has a clear engineering meaning.

3. The synthesis procedure is eventually reduced to some standard problem of H_∞-optimization.

4. The order of the controller obtained as result of synthesis does not exceed that of the initial physical plant. This is important for practical applications.

REFERENCES

Aleksandrov, A.G.; Chestnov, V.N. (1998, a) Synthesis of multivariable systems of prescribed accuracy. I: Use of procedures of LQ-optimization. (English. Russian original), *Autom. Remote Control* 59, No. 7, pp. 973-983; translation from *Avtom. Telemekh*. 1998, No. 7, pp.83-95.

Aleksandrov, A.G.; Chestnov, V.N. (1998, b) Synthesis of multivariable systems of prescribed accuracy. II: Use of procedures of H_∞-optimization. (English. Russian original), *Autom. Remote Control* 59, No. 8, pp.1153-1164; translation from *Avtom. Telemekh*. 1998, No. 8, pp.124-138.

Ackermann J. (1993) *Robust Control: System With Uncertain Physical Parameters*. London: Springer.

Balas G.J., Chiang R.Y., Packard A., Safonov M.G.(2010) *Robust Control Toolbox 3: User's guide*. Natick (Mass.): The MathWorks, Inc..

Barmich B.R. (1994) *New Tools For Robustness Of Linear Systems*. New York. Macmillan.

Bhattacharyya S.P., Keel L.A., Chapellat H. (1995) *Robust Control: The Parametric Approach*. Hertfordshire: Prentice Hall.

Chestnov V.N. (1985) Frequency method for robustness analysis of systems described by differential equations, in Analitical Methods of Controller Synthesis [in Russian], *Collection of Scientific Papers of Higher-Education Shcools*, Saratov Polytechnic Inst., Saratov, pp.50-59.

Chestnov V.N. (1995) An Approach to The problem of synthesizing tolerances for the parameters of linear multivariable systems. *Izv. Ross. Akad. Nauk, Teor. Sist. Upravlen.*, No. 2, pp. 72-79.

Chestnov V. N. (1999) Synthesis of robust controllers of multidimensional systems under parametric uncertainty, based on circular frequency inequalities. *Autom. Remote Control*, Vol. 60, No. 3, pp. 484-491.

Chestnov, V.N. (2011) Synthesizing H_∞-controllers for multidimensional systems with given accuracy and degree of stability. (English. Russian original). *Autom. Remote Control* 72, No. 10, pp. 2161-2175; translation from *Avtom. Telemekh*., No. 10, pp. 170-185.

Doyle J.C. (1982) Analysis Of Feedback Systems With Structured Uncertainties. *IEE Proc*., Vol. 129, Pt. D, No. 6, pp. 242-250.

Doyle J.C. (1983) Synthesis Of Robust Controllers And Filters. *IEEE Proc. 22nd Conf. Decision Contr.*, pp. 109-114.

Doyle J.C., Glover K., Khargonekar P.P., Francis B.A. (1989) State-Space Solution To Standard H_2 And H_∞ Control Problem. *IEEE Trans. Automat. Control*, Vol. 34,No. 8, 831-846.

Farag A., Werner H. (2002) Robust H_2 Controller Design Tuning For The ACC Benchmark Problem And A Real-Time Application. *15th IFAC World Congress*, Barselona, Spain.

Haddad W.M., Collins E.G., Bernstein D.S. (1993) Robust Stability Analysis Using The Small Gain, Circl, Positivity And Popov Theorems: A Comparative Study. *IEEE Trans. Contr. Systems Technology*, Vol. 1., No. 4, pp. 290-293.

Lehtomaki N.A., Sandell N.R., Athans M. (1981) Robustness Results In Linear-Quadratic Gaussian Based Multivariable Control Designs. *IEEE Trans. Automat. Control*, Vol. 26, No. 1, pp. 75–92.

Safonov M.G. (1981) Stability Margins Of Diagonally Perturbed Multivariable Feedback Systems. *Proc. IEEE Conf. Decision and Control.* San Diego. CA, pp. 1472-1478.

Safonov M.G. (1982) Stability Margins Of Diagonally Perturbed Multivariable Feedback Systems. *IEE Proc*., V.129, No . 6, pp. 251-256

Skogestad R.E., Postlethwaite I. (2007) *Multivariable Feedback Control*. N.Y.: John Wiley and Sons.

Zhou K., Doyle J.C., Glover K. (1995) *Robust and Optimal Control*. N J.: Prentice Hall.

Zhou K., Doyle J.C. (1998) *Essentials of Robust Control*. N.J.: Prentice Hall.